APPLIED MECHANICS Reviews

A CRITICAL REVIEW OF THE WORLD LITERATURE IN APPLIED MECHANICS AND RELATED ENGINEERING SCIENCE

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APPLIED MECHANICS REVIEWS

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FLOW PROPERTIES OF BIOLOGICAL FLUIDS

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INTRODUCTION

In regard to their flow properties biological fluids can be considered within three broad categories:

1) Those fluids whose flow properties are critical both to their mode of transport through the body and to their biological functions. These fluids, generally, are naturally occurring solutions that contain either particulate matter or macromolecular material in suspension, which in vivo exhibit highly specific kinds of flow.

2) Those fluids in which the function of flow is negligible or irrelevant, but whose values of viscosity have an influence upon their biologic functions.

3) Those fluids which are man-made solutions of biologically important macromolecules studied under conditions of flow for the purpose of identifying or characterizing their molecular arrangements or other physical properties.

The first category is represented chiefly but not exclusively by blood, blood plasma and various mucus fluids of the body. Examples of fluids of the second category are the cerebrospinal fluid encasing the brain and spinal cord and the fluid within the chamber of the eye. The third category can be represented by dilute solutions of deoxyribonucleic acid DNA in a saline solution prepared from DNA containing cells for the purpose of physio-chemical characterization by such technics as light scattering, sedimentation or viscometry.

For the purposes of this review, only the rheological aspects of the fluids within the first category will be considered. The focus of the presentation is upon the rheological characteristics of the fluid in relation to its physiologic function. This, as will be seen, raises problems that involve virtually all the fields of engineering and, specifically, macromolecular chemistry and physics.

The common rheological features of the important biological fluids to be discussed are as follows:

1) These biologically significant fluids all contain significant amounts of dissolved macromolecular substance, polypeptides, polysaccharides, or both together.

2) Without exception, the fluids are non-Newtonian, and the non-Newtonian features are singularly complicated.

This review will emphasize the fluid mechanical consequences of the non-Newtonian rheology and the interpretation of the origin of the non-Newtonian rheology from the standpoint of the physical and the chemical phenomena involved, with special reference to the chemistry of macromolecular substances.

FLOW PROPERTIES OF BLOOD AND BLOOD PLASMA

Normal blood is a suspension of particulate matter (red cells, white cells, and platelets) in a molecularly continuous medium—the plasma. Plasma, to be described below in detail, consists of a saline solution of three major types of protein: albumin, globulin and fibrinogen. Plasma is separated from the denser particulate matter by centrifugation. The plasma from freshly centrifuged whole blood will clot within 4 to 10 minutes after removal from the body due to the conversion of fibrinogen into fibrin. The fluid remaining after the clot has formed is known as serum and differs from plasma only in the absence of the fibrinogen. To work with plasma, therefore, beyond the period of clotting, anticoagulant substances must be used.

PARTICULATE MATTER IN WHOLE BLOOD

Of the particulate matter, the red cells dominate in volume, occupying on the average 40 vol. per cent* ("hematocrit")

*RELATION OF VOLUME PER CENT TO HEMATOCRIT:

The *hematocrit* (*Hct*) is the volume percentage of red cells and entrained plasma obtained by allowing the cells to settle out, or to be centrifuged, from an initial volume of whole blood prevented from clotting by anticoagulants, in a narrow calibrated tube containing 1 ml of sample. The normal range for human blood is between 40 and 48. A 1-ml sample of normal whole blood would therefore yield a packed bed of cells 0.40 to 0.48 ml in volume, with the clear supernatant fluid (plasma) amounting to a volume of 0.52 to 0.60 ml. Because of the entrained plasma in the bed of cells, the hematocrit is slightly greater than the true volume percentage of cellular material.

of the whole blood. This percentage can go as high as 75 in certain diseases of the heart or lungs, or as low as 10 following acute hemorrhage or chronic blood loss. The human red cell is normally a biconcave disc (a crude analogy would be a tire mounted on an automobile wheel or a doughnut with a very tight plastic envelope that "saucers" in the middle), with a thickness of 2μ and diameter of 8μ. All red cells are of substantially the same size. Since the closest packing of *spheres of the same size* is around 65 volume per cent, blood with a hematocrit of 75 may be on the verge of immobility solely on account of approach to the closest packing of the cells. However, red cells are flexible and capable of distortion into nonsymmetrical shapes (ellipsoids, curled discs) without change in volume. From evidence to be discussed it appears possible that reversible changes of shape of red cells may contribute significantly to the process of blood flow.

As the red cells have a normal density of 1.085 as compared to plasma density of 1.035, they can, for many purposes, be considered as neutrally buoyant particles, yet are removable from plasma by centrifugation.

The white cells are but slightly larger than red cells, and under conditions of normal health are totally insignificant in their volume concentration compared to the red cells, there being 5000 white cells to 5,000,000 red cells per cubic millimeter. Even in acute infection the increased number of white cells does not constitute a significant volume fraction of the blood. For this reason, their effect on blood flow has been largely ignored in the literature. That the white cells can affect the mechanics of blood flow in a critical though unexpected way is suggested in the publications of Fulton and co-workers (1). Their cinemicrographs of the vessels in the cheek pouch of the hamster, during infection or similar stress, show that in the smaller vessels white cells tend to roll along the vessel wall, and that as infection in the animal becomes pronounced, the white cells stick more tenaciously to the wall, and finally form a highly irregular "pavement" of the vessel, reducing its cross section for flow to one third or less of the original. A reasonable model for this situation would consist of a tube of 1-inch inside diameter to the inside wall of which garden peas had been firmly glued, with erratic spacing, and in some places in a double layer. With respect to this model, blood flow would be represented as a 50 volume % suspension of soft flexible peas (red cells) in a sugar syrup. That the drag produced by this combination of rough wall and particulate suspension could be very large is easily comprehended. Quantitative description of the flow dynamics of such a situation in man awaits solution.

The platelets, compared to the white cells and red cells are insignificant in size (0.5μ). Though numerous (400,000 per cubic millimeter) their rheologic importance is probably negligible under normal conditions of health because of the dominance of the red cells. However, it is the platelets (2) that contain thromboplastin, one of the "triggers" of the clotting mechanism. Platelets adsorb on broken tissue or foreign surfaces, their thin envelope bursts releasing the enzyme thromboplastin and the sequence leading to a formed clot begins. Lutz and Fulton (1) demonstrate a generation of platelet emboli in the smaller vessels so as to diminish and finally stop almost completely the blood flow. Occasionally the clot tears away from the vessel wall under the upstream pressure and is carried away as an embolus in the surge of blood through the previously blocked passage. Their fate is not clear although many are filtered out by the vessels of the lung. Significant from the studies of Lutz and Fulton is the experimental observation that new platelet emboli form at the site of the previous emboli.

Plasma: The Continuous Medium of Whole Blood

Human blood plasma, the continuous liquid medium that suspends the particulate substances, consists of electrolytic macromolecules (the plasma proteins) dissolved in an aqueous medium approaching in composition sea water, the average analysis being, per 100 ml of water:

NaCl	860 milligrams
KCl	30
$CaCl_2$	33

with minute but important traces of anions other than chloride, notably the buffering anions: citrate, bicarbonate, phosphate and sulfate.

In this microionic medium are found three major types of protein: *albumin* (3.5 grams per 100 ml plasma); *globulins* (2.5 grams per 100 ml plasma) and the clotting protein, *fibrinogen* (0.5 grams per 100 ml plasma). Molecular weights are respectively 67,000, 150,000, and ca. 340,000 (3). All three proteins are anisometric rigid, negligibly solvated particles having increasing axial ratios as summarized in Table 1. Until recently it was assumed that fibrinogen like albumin and globulin was an elliptical particle, but the electron micrograph studies of Hall and Slayter (4) show fibrinogen to be a double dumbell, as shown in Figure 1.

Plasma also contains widely varying amounts of emulsified fats, cholesterol, free fatty acids, adrenalin, dissolved oxygen and dissolved carbon dioxide, all of which depend on the state of health, period of day, etc. The effects of these substances on plasma rheology has never been carefully examined. It should be noted that most all of the oxygen is

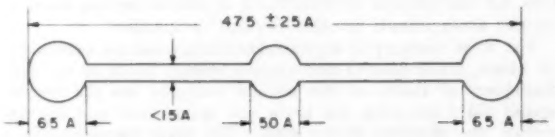

Fig. 1. Dimensions of the fibrinogen molecule, according to Hall and Slayter (4).

TABLE 1

THE PLASMA PROTEINS AND SOME CHARACTERISTICS IN NORMAL HUMAN PLASMA

Protein	Axis lengths as ellipsoids a, Å	b, Å	Mol. wt. M	Average concentration in plasma c, g/100 ml	Number molecules per ml plasma n[1]	Spherical volume of molecule, ml v[2]	Vol. fraction molecules in plasma x_v[3]
Albumin	150	50	67,000	3.5	3.15×10^{17}	1.77×10^{-18}	0.56
Globulin	245	44	150,000	2.5	1.0×10^{17}	7.7×10^{-18}	0.77
Fibrinogen[4]	475	15 to 50	340,000	0.5	0.088×10^{17}	56.0×10^{-18}	0.498

[1] $n = N_A c/(100\,M)$ where $N_A = 6.022 \times 10^{23}$, c = g/dl, M = mol. wt.
[2] v, cm^3 $= \pi a^3/6$ where a = major axis, cm.
[3] x_v, ml. of molecular "spheres" per ml. plasma $= nv$
[4] See Figure 1. Molecule is not an ellipsoid, but calculation of v remains valid.

carried in the blood stream within the red cell where it exists in a loose and reversible combination with the respiratory pigment, hemoglobin. In 100 ml of whole blood, there are 15 grams of hemoglobin and 20 ml of oxygen (measured at 37°C, 760 mm Hg total pressure, saturated with water) bound reversibly with the hemoglobin. The oxygen-carrying capacity of 100 ml of plasma without red cells (and thus hemoglobin) is less than 0.5 ml.

Vessels of the Circulatory System: The Microcirculation

The sequence of vessels in the circulation is given schematically in Figure 2 (5), with data on range of inside diameter, and the pressure over the circuit from the left heart to the right heart. Some comment on the physical characteristics of these vessels is appropriate to the subject of blood flow.

Aorta: The main vessel conducting blood from the heart out to the tissues. In the adult it has a radius of 1.4 cm and the blood therein a mean flow velocity of 35 cm/sec. Blood flow in the major arteries is pulsatile, the vessel dilating during the upstroke of the pulse wave (systole of the heart) and retracting during the relaxation phase of the heart beat (diastole). Blood pressure in the aorta is normally 120 mm Hg during systole and 70 mm Hg during diastole; written as 120/70. The aorta divides into the major arteries.

Artery: Smaller than the aorta, it too has a heavy muscular wall ranging from 0.5 to 2 mm in thickness and is encased in a fibrous sheath. Being elastic it conducts a pulse wave (both in the fluid and in the vessel wall) like the aorta. The pressure in the arteries is essentially the same as that in the aorta.

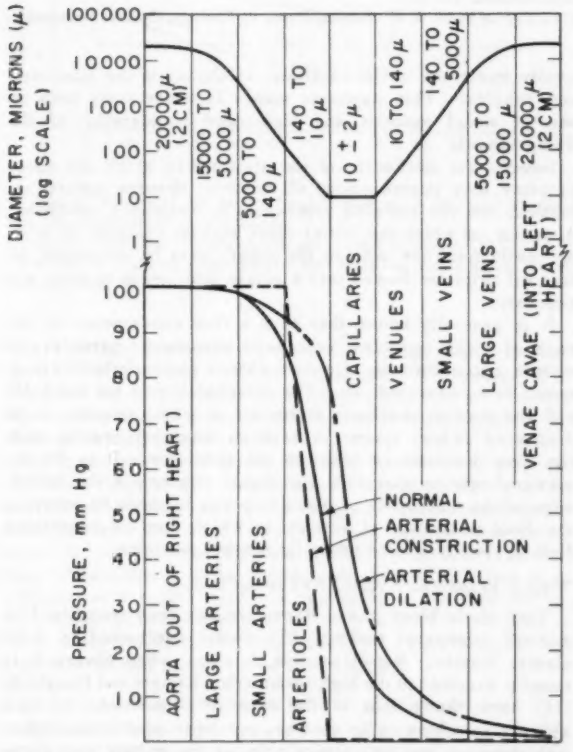

Fig. 2. Pressure as a function of type of vessel and diameters of the vessels of the circulatory system.
Reproduced through the courtesy of A. C. Guyton (5) and W. B. Saunders Co.

Arteriole: A smaller vessel than the artery with widely varying sizes but still by definition containing a muscular wall. The diameter varies from 500 to 10μ. As the arteriole gets smaller the pressure wave decreases. The major pressure drop throughout the vascular system is in these vessels, so that flow in the terminal arteriole is steady at a pressure of approximately 20–50 mm Hg. The arterioles are constantly dividing into smaller vessels until they reach the capillaries. *The arterioles are richly supplied with nerves controlling the tone or diameter of the vessels* with a range permitting complete closure or dilation up to 4 times the normal diameters.

Capillaries: These are the smallest vessels with very thin tissue walls devoid of muscle. They are 8 to 9μ in diameter and probably are 70–100μ in length. All metabolic exchange processes occur across the wall of this vessel. There is no nervous control of the vessel diameter. In many animals there is a sphincteric, or valve function, at the entrance to the capillary. The red cell is essentially the same diameter as the capillary. The capillaries empty into the enlarging venules, thin-walled vessels equivalent to the arterioles but with low pressure.

Venules and Veins: The return conduit of low pressure is thin-walled. To prevent back flow in the extremities of erect animals a system of valves within the vessels are operative. The deoxygenated blood courses through enlarging vessels to enter the major veins, the *venae cavae* which empty into the heart. The deoxygenated blood passes from right atrium to the right ventricle, (pressure 25/0) whence it is pumped into the pulmonary artery, (25/8) thence into the artery, arterioles and capillaries of the lung. The air sacs of the lung (alveoli) exchange oxygen and carbon dioxide with this blood and the blood now resaturated with oxygen passes through the pulmonary veins to the left atrium of the heart to the left ventricle, (pressure 120/0) and thence out into the aorta.

While these facts are well accepted by physiologists, certain conclusions that follow from them have not received adequate attention from the standpoint of their rheological consequences. Two aspects in particular are noteworthy:

1) The major pressure drop in the blood flow from the aorta to the return flow in the right heart occurs in the "*microcirculation*" consisting of *arterioles*, *capillaries* and *venules*. In these small vessels 80% of the total pressure drop occurs, and of this 80% and under normal conditions at least a half lies in the arterioles, another 40% in the capillaries and the remaining 10% in the venules. Figure 2 also shows how, according to where there is vasoconstriction or vasodilation, the pressure drop occurs predominantly in the arterioles or in the capillaries. These are normal conditions.

2) In the vessels of the microcirculation, the maximum vessel diameter is about 18 times that of an individual red cell and diminishes in the capillary, where oxygen exchange and all other exchange processes occur, to about 1.1 times the red cell diameter. Red cells pass through the capillaries in single file from all photographic evidence (1) so far obtained on the tissues of living animals and by observation of the capillary bed in the human eye (6).

Processes of Flow in the Microcirculation

From the observations cited above, it seems reasonable to assert that the part of the entire vessel circuit that effectively controls the pressure drop from one side of the heart to the other is the microcirculation, and not the larger vessels. As the ratio of the red cell diameter to that of its containing vessel is between 1.1 and 18 in the microcirculation, and as in the white cell "pavement" of severe infection the wall "roughness" is of the same size as the particles that must be conveyed, it seems clear to the reviewers that blood cannot be considered as, and should not be tested as, a homogeneous fluid in connection with study of flow in the microcirculation. This is supported by the classic work of Fahræus and Lindquist (7) on the viscosity of blood as a function of diameter

of the glass tube in which it was tested. The viscosity was found to decrease greatly as the ratio of tube diameter to red cell diameter decreased below about 20. It is interesting to note parenthetically how difficult in general it is to model a process of nature: Though in the living circulation red cells must go through capillaries with a diameter ratio of around 1.1, Fahræus and Lindquist (7) found that their capillary viscometers became totally obstructed (viscosity became infinite) when the bore of the tube was 5 red cell diameters or less. Similar results have since been noted by Mason and co-workers (8). The plugging of the glass tube can be ascribed tentatively to a number of factors, the most likely of which is strong adsorption of the red cells and plasma elements on glass.

A process of interest to hydrodynamicists is the "axial drift" of suspended particles in laminar flow under a nonuniform velocity gradient, specifically Poiseuillian-type flow in a tube. Axial drift of red cells away from the wall toward the axis of a vessel has been argued either as an experimentally observed fact, or as a possibility, by physiologists. If this process should occur, clearly the cell free layer of plasma remaining at the wall would serve as a lubricious medium and reduce the drag compared to that expected if the blood were to remain homogeneous.

At present it is impossible to give a final account of whether axial drift is relevant to blood flow. Saffman (9) theoretically predicts axial drift of any single particles in Poiseuillian flow. On the contrary Mason (8) experimentally showed that rigid spheres and discs do not undergo axial drift, but that deformable particles including deformable spheres and discs do undergo axial drift. Coulter and Pappenheimer (10) report that they have demonstrated axial streaming of blood cells by change of electrical resistance caused by flow, whereas Bayliss (11) amends earlier reviews of the subject (12, 13) in concluding that no firm evidence can be adduced to prove the existence of axial drift, or the necessity of axial drift to explain anomalous flow. Furthermore, it is most likely that an excluded volume effect similar to that proposed by Vand (14) may be operative. Briefly, in Vand's theory relating to spheres in a suspension, since the center of any sphere cannot come closer to the wall than a distance equal to its radius, there will be a layer at the wall having a lesser volume percentage of suspended spheres on the average than the main bulk of the suspension—the difference becoming increasingly significant as the volume percentage of suspended spheres increases. After repeated observation of the cinemicrographic studies of hamster microcirculation by Fulton (1), the reviewers cannot report their conviction that axial drift is operative in the microcirculation.

In the hamster capillaries the red cells are seen (1) to be lined up transversely in single file like a stack of wafers, *but separated from each other by plasma.* Figure 3 is one frame of the cinemicrographic studies at 900 ×, reproduced through the courtesy of Dr. Fulton, showing this effect. The cells move through the capillary at a velocity of 10 to 20 cell diameters/second. Since the cells are already at the axis, and in single file, neither the Vand wall layer nor the axial drift theory is relevant. In the arteriole preceding the capillary, and in the venules following the capillary the Fulton studies reveal a flow that is impressively granular in nature, in which the red cells as observed next to the vessel wall bend, rotate, tumble and exchange places in a chaotic manner. If one grants a hematocrit of 40 to 50 in the hamster, as in the human, with the possibility of elastic deformation of the cells under flow, it is difficult to visualize how hydrodynamic forces responsible for axial drift would be effective in concentrating an already closely packed suspension of deformable cells. The hamster red cell is the same diameter as that of man. The cinemicrographic studies by Fulton, at magnifications of 200 to 900 ×, are interpreted by the reviewers as generally supporting the view that axial drift,

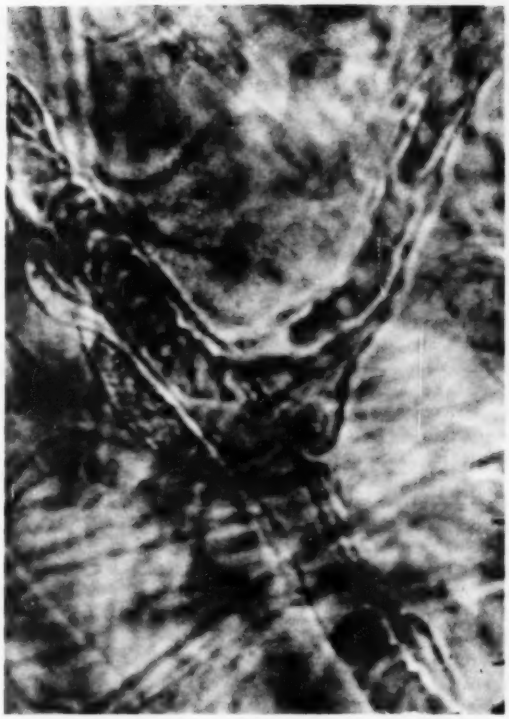

Fig. 3. Flow of red cells through a capillary in the cheek pouch of a hamster, 900 ×.
Courtesy of Prof. G. P. Fulton, Dept. of Biology, Boston University.

totally irrelevant in the capillary, is absent in the arterioles and venules. This important matter is under study both by use of model systems and high-speed photography of the living vessel.

Indeed, the mechanics of the arteriole in which the major pressure drop occurs among all vessels, deserve special attention, for the arteriole represents a bottleneck condition. A stream, of which the initial cross section contains 18 or so red cells from one side to the other, must be converged so that all cells are formed into a single line, ready to enter the capillary.

It is generally known that such a flow convergence in the case of RIGID particles, especially anisometric particles, in volume concentrations of around 45% is particularly likely to result in an impacted jam. The deformability of the red cells and the special rheologic properties of whole plasma, to be discussed below, appear to have an important bearing upon the flow dynamics of blood in the arterioles. It is the reviewers' opinion that very significant progress in the knowledge of the mechanics of blood flow can be made by studying the fluid mechanics of systems in which there is converging flow of closely spaced neutrally buoyant particles.

WHOLE BLOOD AS A NON-NEWTONIAN LIQUID

That whole blood is non-Newtonian has been recognized by a small number of workers (15), though overlooked by a far greater number. Non-Newtonian rheology, when observed, is usually ascribed to the high hematocrit. Krieger and Dougherty (16) have shown that in the case of suspensions of rigid spheres in a Newtonian medium, non-Newtonian (pseudoplastic) rheology becomes appreciable as the volume concentration of spheres is increased above about 30 per cent. The collision mechanics leading to non-Newtonian flow of concentrated suspensions of spheres would in general, with amenda-

tion to allow for shape and orientation, be applicable to suspensions of discs, and therefore red cells.

The first comprehensive study of the effect of hematocrit on non-Newtonian rheology of whole blood appears to be that of Wells and Merrill (17). Their results are summarized in Figure 4. The shear rate dependence of the *apparent viscosity* (defined as the ratio of point values of shear stress to shear rate) with shear rate appears from this work (17) to be far more profound than elsewhere reported in the literature. This is ascribed to the fact that in a special cone and plate viscometer operating on small volume samples, freshly drawn whole blood was viscometrically tested before clotting, *without* the addition of standard anticoagulants (acid citrate dextrose ("ACD"), heparin, or oxalate). Further work on whole blood by the same authors (18) showed that addition of anticoagulants (with no change in hematocrit) lowered the general level of viscosity and caused the blood to approach a Newtonian flow (less variation of viscosity with shear rate).

The implications of Figure 4 appear to be that in the larger vessels of the circulation, in which it is relevant to consider blood as a homogeneous liquid, marked deviation from Poiseuil-

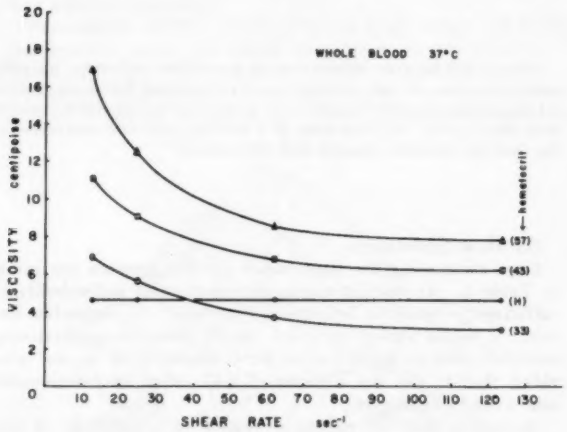

Fig. 4. Viscosity of whole *fresh* blood as a function of shear rate for three values of hematocrit.
H refers to National Bureau of Standards calibration oil "H."
Courtesy of *Science*, published by the American Association for the Advancement of *Science*.

lian flow may occur, in the direction of plug flow. In other words, the parabolic velocity profile may be replaced by a flat-front velocity profile, there being uniform velocity across most of the vessel cross section with substantially all of the shear occurring near the wall.

At least one result of the flat-front profile is to make the residence time in the larger vessels of all elements of the flow more uniform. In Poiseuillian flow the velocity at the axis is twice the mean velocity which occurs on a radius equal to two-thirds of the tube radius. Thus the residence time of an element at the center of the tube is half that of an element on a radius equal to two-thirds of the tube radius. The shear rate dependence of blood viscosity is such as to make these values more nearly equal. The physiological significance of this has yet to be established.

Plasma as a Non-Newtonian Liquid—Experimental

In view of the importance of the plasma rheology in the vessels of the microcirculation it is surprising to find (a) the lack of much prior work on plasma rheology and (b) the erroneous conclusions that have, in good faith, been drawn from the limited data available. There appears to be no prior disclosure in the literature that blood plasma is a non-Newtonian liquid. Only one report (21), to be discussed, has been found concerning blood serum (plasma having its fibrinogen removed) that shows a shear dependent viscosity. That it is, in fact, a non-Newtonian fluid has recently been shown (18), by means of a cone-plate viscometer (19) designed to obtain measurements on *freshly drawn* untreated plasma (cellular matter centrifuged out) before clotting occurs. Typical data are shown in Figure 5. The viscosity of untreated fresh plasma, top curve, is seen to increase with decrease of shear rate, below ca. 100 inverse seconds. The points were reproducible over the three to four minutes of time available before clotting occurred.

Measurements by the same viscometer on plasma drawn from the same subject but treated with the anticoagulant heparin showed that the shear rate dependence of viscosity was reduced, though not eliminated, and that the whole level of viscosity was somewhat lower than that of the untreated plasma. (Middle curve Figure 5). Plasma drawn from the same subject and treated with oxalate showed no dependence of viscosity on shear rate, and a still lower level of viscosity.

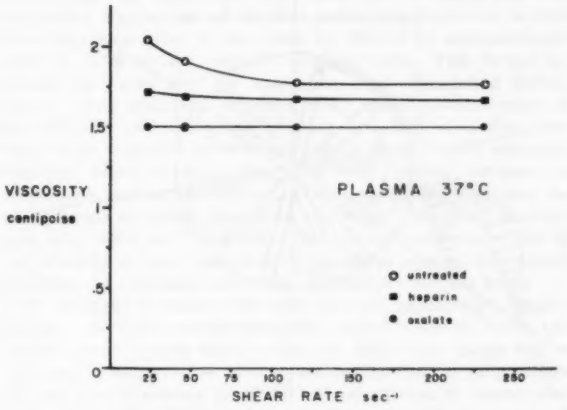

Fig. 5. Viscosity of plasma as a function of shear rate; fresh plasma, plasma with heparin, and plasma with oxalate.
Courtesy of Science.

The commonly used "acid citrate dextrose," ACD, has been found to have the same effect as oxalate.

There appear to be several reasons why the non-Newtonian character of blood plasma eluded detection in prior studies. In the work of Brundage (20) with a coaxial cylinder viscometer, the sample was oxalate treated, and its viscosity at two different shear rates was found to be identical.

Kreuzer (21), testing blood SERUMS in capillary viscometers under very high shear rates, 1000 to 20,000 sec^{-1}, observed non-Newtonian behavior. The viscosity levelled off to a constant value about 1.5 times that of water, above about 3000 sec^{-1} of shear rate. The lower limit (1000 sec^{-1}) of shear rate accessible to Kreuzer appears to be the upper limit of shear rate attainable in any part of the *in vivo* circulation. Comparing the plateau in viscosity from 50 to 120 sec^{-1} in Figure 5 for blood *plasma* with the plateau noted in blood *serum* by Kreuzer leads one to the speculation that there may be *two* different constant levels of viscosity in blood plasma pertinent to different ranges of shear rate separated by one or two decades. Mechanisms to account for two plateaus are discussed below.

Bingham (22), whose pioneering rheological studies led to the naming of one category of non-Newtonian fluids as *Bingham plastics*, observed no evidence of non-Newtonian character-

istics in studies on mixtures of *oxalate-containing* plasma proteins.

Capillary experiments on plasma and serum other than those cited above were conducted in such a way as to obtain a flow time at only one pressure (or capillary length, or radius). Thus, only one value of shear stress versus shear rate was obtained, from which of course nothing can be inferred regarding the rheological character of untreated plasma.

Plasma as a Non-Newtonian Liquid—Theoretical

There appear to be three possible origins of non-Newtonian rheology in whole plasma: (a) orientation of the anisometric protein molecules in the direction of shear; (b) network association of the protein molecules; and (c) orientation and association acting concomitantly.

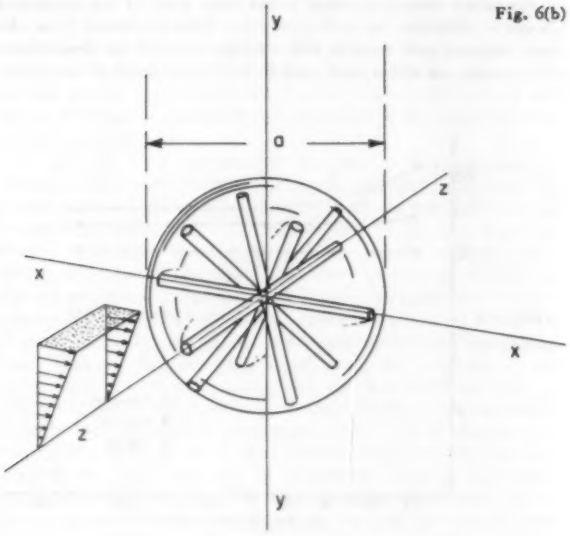

Fig. 6(b)

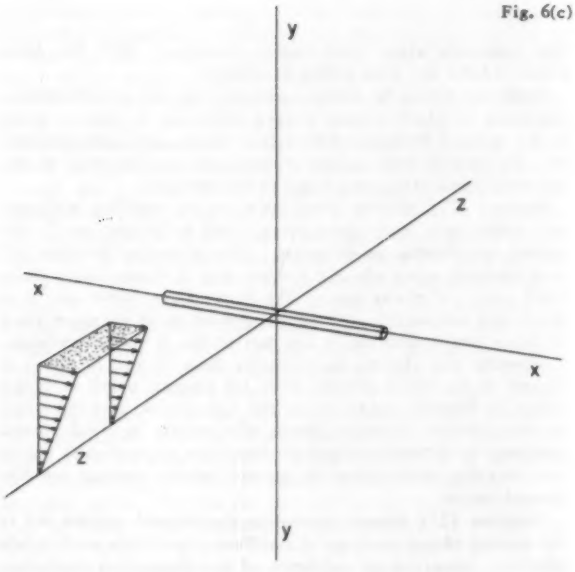

Fig. 6(c)

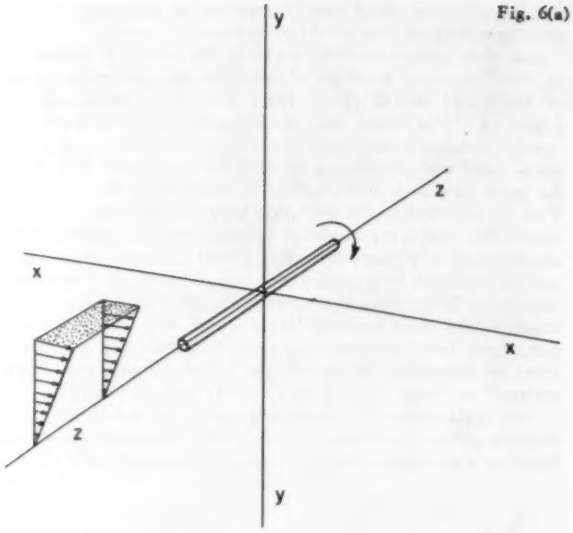

Fig. 6(a)

Fig. 6. (a) Random orientation of a rod-like molecule, showing spherical locus of end positions under rotational Brownian motion. (b) Alignment of a rod-like molecule in the flow (x) direction (coaxial with the x axis). (c) Alignment of a rod-like molecule transverse to the flow (x) direction (coaxial with the x axis).

(a) *Shear Orientation*

Data concerning the three major plasma proteins are given in Table 1. As each of these macromolecules individually is sufficiently small to be affected violently by Brownian motion, it would rotate in space in all possible orientations, provided that no other forces were operative on it, and provided that it was not associated with other macromolecules into a larger aggregate.

Assuming that the volume occupied by a molecule is that of a sphere (Figure 6a) swept out in a diameter *a*, where *a* is the major axial length of the molecule, one may compute in conjunction with the concentration of the protein species, the fraction of the total volume of plasma that is occupied by the spherical volumes of the protein molecules. The results are shown in Table 1. It is remarkable that for each species, the fraction computed (ca. 0.6) is nearly that for closest packing of spheres.* It is then evident that the summation of the spherical "volumes of action" for all three proteins is roughly 3-fold greater than the closest packing of spheres. This can mean only that interpenetration of spheres of action occurs (if one continues to grant that the protein molecules move independently of each other).

As reviewed in detail by Burgers (23), the orientation of ellipsoidal or rodlike particles under a shear field produces a significant decrease in viscosity, to ultimate values that differ according to whether the particle becomes aligned in the flow direction (Figure 6b) or in the *x*-axis transverse to the flow direction (Figure 6c).

Under the conditions of crowding of the spheres of action of plasma proteins in human plasma, one therefore anticipates a greater reduction of viscosity with increase of shear rate than expected solely from the orientation effect, because additionally the interference between protein molecules would be reduced.

*We are indebted to Dr. D. F. Waugh, Prof. of Biophysics, Dept. of Biology, M.I.T., for calling our attention to these facts.

(b) *Association of Protein Molecules into Networks*

Entirely apart from shear orientation effects, non-Newtonian rheology can arise in a suspension of particles as a result of aggregation of the particles into networks, via electrostatic, ion-bridge, or hydrogen bonding sites, that collapse as shear increases, and reform as shear decreases. A theory of non-Newtonian rheology based on network formation-destruction was proposed by Goodeve (24) and amended by Storey (25) to suit the case that the suspended particles are in fact micromolecules, thus smaller by two or three orders of magnitude than those treated by Goodeve. A consequence of the model formulated by Storey was that even *spherical* particles participating in network aggregates could lead to non-Newtonian rheology. This was experimentally demonstrated in solutions of the starch component amylopectin—an approximately spherical, rigid, polysaccharide macromolecule capable of association by multiple hydrogen bonds.

Association of anisometric rigid macromolecules into aggregates in which there is no general alignment, as suggested in Figure 7, would lead to structures in which the continuous medium was sequestered in some degree. This would lead to a higher effective volume fraction of the suspended phase and thus a higher viscosity.

Association of the plasma proteins might occur via hydrogen bonding sites but could more likely occur via a Ca^{++} bridge between $-COO^{-}$ groups in neighboring molecules.

Network aggregation of macromolecules does not always lead to a change of viscosity. Doty (26) studying polyvinyl chloride in dioxane solutions showed that the particles varied in size from single macromolecules to clusters of perhaps hundreds without significantly changing the intrinsic viscosity, whereas all other size characterizing properties, such as sedimentation rate, turbidity, etc., changed radically. More recently W. Kuhn and P. Moser (27) have shown that an aggregation of macromolecules can produce, depending on the manner of assemblage, either an increase, a decrease, or no change in the viscosity for a given, fixed concentration of macromolecules in the solution.

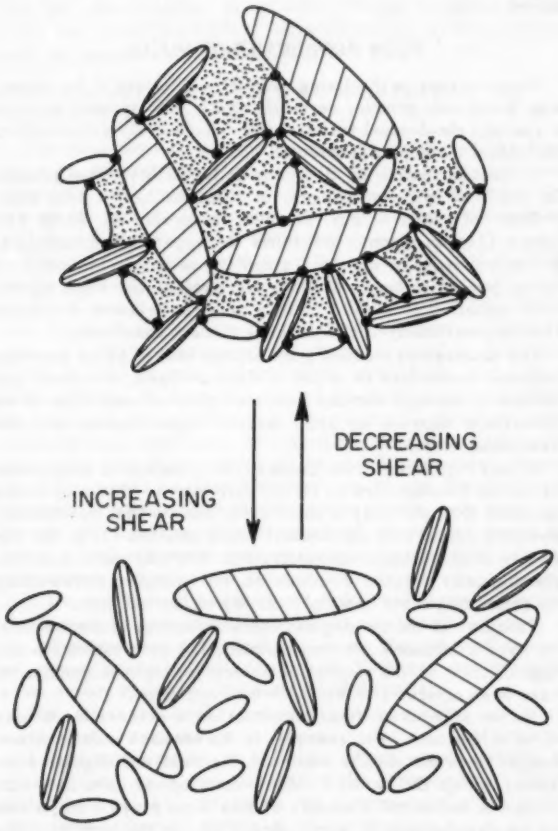

Fig. 7. Reversible aggregation-disaggregation of anisometric molecules under varying levels of shear. Solid dots = active sites of association. Stippled background represents entrained liquid.

(c) *Orientation Effects and Aggregation Effects Operating Concomitantly*

That orientation by shear of anisometric rigid macromolecules in solution could affect the size and character of aggregates formed therefrom is discussed by Storey (25). The work of Joly (28) on solutions of serum albumin suggests that both aggregation and orientation effects may be important in plasma rheology. Joly measured the extinction angle of horse serum albumin suspended in various electrolytes in a coaxial cylinder device, as a function of shear rate. (No provision was made for obtaining shear stress, and thus no measurement of viscosity as a function of shear rate was obtained). Joly inferred from the extinction angle data that as shear rate was *increased*, aggregates of albumin molecules *increased* in size to a maximum size of the order of 5000 Å at approximately 1000 to 2000 inverse seconds of shear rate. With further increase of shear rate the aggregate size diminished indefinitely. Joly therefore proposed that, initially, orientation of the albumin molecules under shear occurred such that they were more disposed to form aggregates than in their randomly oriented state at rest, but that, with further increase of shear rate, mechanical stress on the aggregates disrupted the weak bonds by which they held together. The level of shear rate (ca. 1000 sec^{-1}) at which Joly found maximum effect is conceivably a level pertinent to the shear rate in the plasma between the circumference of an erythrocyte and the wall.

In the light of these three mechanisms (orientation, aggregation, and their combination) the experiments of Wells and Merrill (18) suggest that in the low shear rate range (up to 250 sec^{-1}) aggregation effects are predominantly responsible for the non-Newtonian rheology. If orientation of macromolecules in a shear field occurred in this range of shear rate, it should not be possible to eliminate the effect by adding citrate or oxalate. If one grants this premise, then the drop from the first plateau in the viscosity shear rate curve (that found by Wells and Merrill) to the second found by Kreuzer (21) is to be ascribed to orientation of macromolecules in a shear field.

Significance of Non-Newtonian Plasma Rheology

We now consider the effect of the experimentally observed shear rate dependence of plasma viscosity on flow in the capillaries, and specifically we refer to some of the results in the previously discussed studies of Fulton (1) using cinemicography on the capillaries in the hamster cheek pouch. The cinemicrograph of Fulton, Figure 3, has been schematically interpreted in Figure 8, to emphasize the role of geometry and flow velocity. The flow velocity of the red cells was of the order of 10 capillary diameters (D_c) per second. If we assume that the red cell diameter is about 0.8 capillary diameters (0.80 D_c) (possibly at times as high as 0.98), the radial clearance between red cell and capillary wall would be about 0.10 D_c. Thus, the velocity gradient in the plasma between the red cell and the capillary wall, estimated as the red cell velocity divided by radial clearance, would be $10\ D_c/0.1\ D_c = 100\ sec.^{-1}$. (The difficulty of obtaining an exact measurement of the diameters of capillary and red cell during the observed flow precludes a rigorously accurate determination of the velocity gradient, but the above is probably correct in order of magnitude).

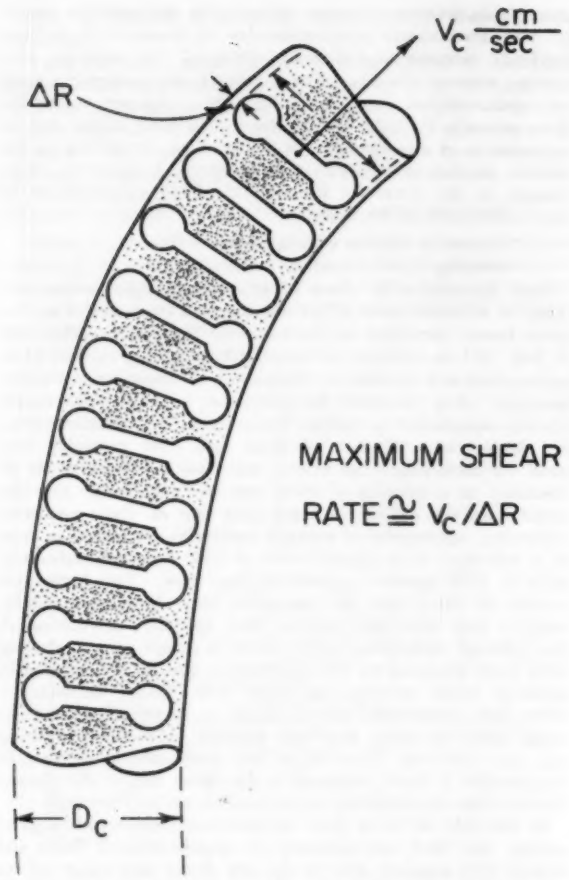

Fig. 8. Schematic diagram of red cell flow through a capillary; plasma under high shear rate between cell and wall, very low shear rate between any two red cells (stippled on drawing).

Between the flat surfaces of any two adjacent cells in the pictures of Fulton and Lutz, i.e., in the "core," negligible relative motion was observed. The shear rate would appear to be of the order of 1 inverse second or less at the central axis.

Comparison of these estimates of shear rate in flow through a capillary with the uppermost curve of Figure 5 (untreated human plasma viscosity versus shear rate) shows that the layer of plasma *between* red cell and wall should have reached its minimum viscosity whereas the plasma separating one red cell from the next could be of substantially greater viscosity. Such a condition would presumably lead to hydrodynamic stabilization of the red cells as they proceed through the capillary, i.e., preventing rotation.

Considering that the prime function of the red cell is the transport of oxygen to the tissues, one may theorize that such a stabilized orientation of the red cells may have broad physiological significance. For example, this orientation maximizes the effective red cell surface presented to the capillary walls, minimizes the distance of this surface from the capillary wall, and thereby, maximizes the partial pressure gradient of oxygen for any given difference of partial pressures. Concomitantly, these effects (area and gradient maximized) should produce the greatest rate of oxygen transfer between any red cell and the wall. Similarly, the high shear rate imposed on the plasma at the wall may have some bearing upon the dynamics of the membrane exchange. Most membrane studies have been conducted as static test systems (29). The diffusion coefficient of a diffusing solute in a liquid medium varies inversely as the viscosity (30), according to the Stokes-Einstein equation:

$$D = \frac{kT}{6\pi r\eta}$$

D = diffusion coefficient, cm²/sec
k = Boltzmann constant
T = absolute temperature, °K
r = radius of diffusing molecule, cm
η = viscosity of liquid phase, poise

It is possible that the diffusion coefficient in a non-Newtonian liquid varies with shear rate in a manner opposite to that of the viscosity. Thus, decreasing viscosity by increasing shear rate might have the effect of increasing the diffusion coefficient. In the capillary, therefore, by this line of speculation, the maximum value of diffusion coefficient would be achieved between the capillary wall and the perimeter of the red cell. The influence or application of this principle to gas exchange in the capillaries of the tissues or the lungs has received little attention in the medical arena. This may be due in part to the rarity of clear-cut definitions of plasma and blood rheology in the medical literature. It is the reviewers' opinion that the viscosity effect on diffusion of both solute and gases is highly critical to the exchange rates across the capillary wall and that these matters deserve comprehensive and detailed study.

FLOW PROPERTIES OF MUCUS

Mucus occurs in the living system in a variety of locations, e.g. bronchial, gastric, cervical. Each form appears to have a specific rheological purpose, and consequently a distinctive, individual rheological character.

A common feature of all mucus is its ability, when present in small weight percentages, to "immobilize" a very large volume of water. For example, according to White and Elmes (31), in chronic bronchitis the expectorated mucus, at a concentration of around 1 gram/dl., is a gel, incapable of being passed through capillary viscometer, and when diluted with agitation to a third of this value it becomes a viscous fluid approximately 100 times more viscous than water.

The rheological problem goes beyond that of blood rheology because mucus may be either a viscous liquid, an elastic gel of tensile strength varying from negligible to very high, or an ill-defined mixture of both, usually contaminated with adventitious matter.

A very significant impediment to the quantitative description of mucus rheology lies in (1) the difficulty of obtaining intact samples from the body without manipulation that unavoidably destroys its *in vivo* rheological character, and (2) in the difficulty of designing a test instrument that will yield absolute, rheologically definitive values of, for example, stress-strain on gelled fragments of widely varying shape and size.

Problems in the rheological characterization of mucus under *in vivo* conditions (in contradistinction to purification and high dilution for the purpose of molecular weight determination) have been reviewed by Wells, Denton and Merrill (19).

Mucus appears to function either as a defense mechanism or as a lubricant. For example, in the stomach it is a critical barrier between muscle wall and the acid and digestive enzymes within the cavity. Gastroenterologists have long suspected a decreased viscosity of this layer plays a major role in the development of peptic ulcer (38). In the bronchi of the lungs the normally thin mucus plays a role in the defenses against inhaled foreign matter, in the functions of hair-like (cilia) projections from the lining cells and in the air-conditioning function of the upper airway (39).

Chemical Composition of Mucus

The macromolecules that constitute mucus are just opposite in nature to the blood plasma proteins.

The plasma proteins are rigid; the mucus macromolecules are flexible, random-coiling. The plasma proteins are negligibly solvated by or permeated by water; the mucus macromolecules form highly swollen, water-sequestering coils that vary in size substantially according to their precise ionic environment. The plasma proteins are anisometric, retaining their *shape* under all conditions of shear stress; the mucus macromolecules are easily distorted from the symmetrical random coiling shape at rest into ellipsoidal, solvent-permeated masses of variable axial ratio. The plasma protein molecule, even fibrinogen, makes a relatively small contribution to the whole viscosity of the solution, unless associated with other macromolecules, whereas the *random coiling* mucus macromolecule of equal molecular weight will in general make an impressive contribution to the whole viscosity.

For some time it has been recognized that, in fact, mucus consists of two kinds of macromolecules, a mucopolysaccharide, hereafter abbreviated *MPS* and a mucoprotein hereafter abbreviated *MP*, both the MPS and MP being random coiling macromolecules.

Kwart and Shashoua (32) have provided the most chemically detailed account of the interaction whereby MPS and MP are linked to form mucus. Essential features on the macromolecules are: carboxylate ions (COO^-) on the protein and hexosamine —$\underset{|}{C}HNH_3$ on the MPS. Calcium ion Ca^{++} and sulfate ion $SO_4^=$ are involved in an *ionic linkage* between the MP COO^- and the MPS > $CHNH_3^+$ while competing for these groups with the monovalent anions and cations, Na^+, K^+, Cl^-.

Ionic Strength, Hydrogen Ion Concentration, and Flow Properties of Mucus

The structure and linkage proposed by Kwart and Shashoua at once suggest that three general factors in the ionic environment may be involved in the rheological character of the mucus: the total ionic strength, the hydrogen ion concentration, and the concentration of divalent ions.

The general trends to be expected by variation of the total ionic strength (total equivalents/ml of all ions: Na^+, K^+, Ca^{++}, H^+, etc.) are well comprised in the treatise by Conway and Dobry-Duclaux (33). Here it is sufficient to point out that, as ionic strength decreases, the random coiling molecules (MPS or MP) expand, become more voluminous, and entrain more solvent. If the experiment of diminishing ionic strength were carried out in vitro by dilution of mucus with pure water to vanishingly small concentrations, the macromolecules would eventually extend into semi-rigid threads. Solvent-sequestering capacity would be lost but, because of the possibility of orientation of the rigid threads under shear, very great decrease of viscosity with increase of shear rate would be noted. Probably under most *in vivo* conditions, high dilution with absence of microions cannot occur.

In the presence of a significant hydrogen ion concentration, the carboxylic groups on the aspartic or glutamic residues of the MP must cease to be ionized. Hence they become capable of the characteristically tenacious hydrogen bonding with other carboxyls (and with hydroxyls of the MPS).

The effect of strong hydrogen bonding, intra- and intermolecularly, is to produce variously swollen gel, a precipitated and desolvated phase, or a fibrous phase—none capable of viscous flow, as is well attested by the experiments of Kwart and Shashoua (32) and others on synthetic polyelectrolytes containing carboxylic groups in an acid medium.

Divalent Ions and Flow Properties of Mucus

Perhaps the most subtle factor in mucus rheology is the concentration of divalent ions, primarily calcium and sulfate. If one accepts the hypothesis of Kwart and Shashoua that *both* are necessary for the formation of a link between the hexosamine on the MPS and a carboxylic ion on MP, then the number of effective bonds formed depends quite critically both on the Ca^{++} and also on $SO_4^=$ concentration, as well as on the probability of favorable conformation between segments of neighboring MPS and MP. It is especially to be noted that an excess of Ca^{++} could separately saturate hexosamine groups on MPS so as to *prevent* association between MP and MPS. Kwart and Shashoua observed solubilization of a gel into a viscous fluid by placing bronchial mucus in strong $CaCl_2$. These factors would argue for a much more discriminating linkage between MPS and MS through divalent ions than via carboxylic acid hydrogen bonds in an acid medium.

So far it has been tacitly implied that all mucoproteins MP and all mucopolysaccharides MPS are identical chemically regardless of their origin in the human system. This is most probably an erroneous assumption. At present, the characterization of MP and MPS from various physiological sites is so incomplete that no conclusions can be drawn. However some insight can still be gained into the rheological function by considering the variation of ionic environment on standard MP and MPS.

Physiologically Important Mucus Materials

a) *Cervical Mucus*

The mucus obtained from the cervical canal of the uterus is both physiologically and rheologically significant. Cervical mucus normally is a liquid of relatively low viscosity which becomes more viscous (a gel) during pregnancy. It has been argued that this is a natural defense mechanism to prevent ingress of sperm during this period and that high viscosity at other times may play a role in problems of sterility. Neurogenic and hormonal influences probably trigger changes in the microionic environment of the MPS and MP in the cervix, specifically through a calcium ion or sulfate ion variation. The problem of infertility makes this a worthwhile area for more intensive study. The paper of Viergiver and Pommerenke (40) is representative of the studies upon cervical mucus viscosity in relation to ovulation and other factors. It would appear that the rheological character of cervical mucus is under delicate control probably through hormonal mechanisms mediated via changes in the ionic concentrations within the cervical glands. From the papers on this subject by Gibbons and Glover (34), Beller and Vogler (35), and Scott-Blair and Glover (36), the reviewers believe that important future advances might be made in this area by a careful theoretical and experimental investigation of the interaction between oppositely charged polyelectrolytes, with and without divalent microions. Considerable study has been carried out by the gynecologists on these matters, principally in relation to the influence hormones have upon the gel-sol transformations that occur within the cervical mucus. The point to be made is that the rheological behavior of cervical mucus has a direct and critical bearing upon its normal and abnormal physiologic functions.

b) *Gastric Mucus*

It is well established that the stomach of man is lined by a thick coating of viscous mucus, whose adherence to the underlying cells permits it to resist the dislodging contractions of the stomach wall. This mucus layer and the cell lining beneath form a strong barrier protecting the stomach wall itself against the digestive juices and acid fluids (pH less than 3.5) within the stomach cavity (38). Wells and Kyle (41) list the defense functions of the mucus layer as the absorptive powers for the enzyme pepsin, the buffering powers against acid, the viscosity which resists flow and the cohesiveness that allows the mucus layer to be continuous. A comprehensive description of these gastric secretory functions is given in the text by Babkin (42).

In the acid environment of the stomach it is tempting to speculate that strong hydrogen bonding due to the undissociated carboxylic acid groups on MP may be responsible for maintaining the tenacity and viscosity of the mucus lining. Although many studies of the viscosity of gastric mucus have been made under a variety of conditions the procedure remains a valuable tool in the analysis of factors altering the defense and digestive mechanisms of this fluid within the stomach.

c) *Bronchial Mucus*

The airways of the lung are closely analogous to a large tree held upside down by its main trunk. Inspired air from the pharynx passes down through the main bronchus (tree trunk), into smaller bronchi (main branches off the tree trunk), thence into the bronchioles (smaller branches), thence into the respiratory bronchioles (stems of leaves) and finally into the atria which are encompassed by alveoli (leaves). Atria and alveoli correspond in the analogy to the leaf tissue in the tree.

Bronchial mucus is given off by the glands in the walls of the bronchi and bronchioles and is continually worked upwards toward the glottis (the division between bronchus and esophagus) by the action of the hair-like projection (cilia) atop each lining cell. In the standing or sitting human this upward flow occurs against gravity. A major function of the movement of bronchial mucus is to keep the air passages clean (by sweeping out inspired dust or foreign matter). Mucus also has the important function of holding water as a lining at the wall of every part of the airway system. Evaporation of water from this lining is a crucial part of the continuous temperature-regulating process of the body, and as an emergency measure is effective in minimizing damage to tissue by accidental inhalation of hot gases (39). Dehydration of bronchial mucus, or its conversion into a tenacious gel, accompanies severe respiratory disorders, such as bronchitis or during the use of tracheostomy i.e., a short-circuiting tube placed directly through the wall of the trachea in the neck.

The cilia, by which mucus is propelled, are hair-like projections from the walls of the airways, and operate with a rhythmical whiplike motion. The cilia usually are completely immersed in the layer of mucus being propelled. The cilia have a rapid backward (toward the foot) motion through their mucus overburden, followed by a slow forward (toward the head) stroke. The viscosity of normal bronchial mucus has been demonstrated by Wells, Denton and Merrill (19) to decrease greatly under shear. Thus mucus acts "viscous" during the forward stroke of the cilia, but "thin" during the backlash. This rheological feature may account for the effective anti-gravity progression of mucus into the glottis in the maintenance of clear airways.

That the microionic environment of bronchial mucus is most likely responsible for its fluidity is attested to by the chronically gelled mucus in lungs of persons afflicted with cystic fibrosis. In these patients it is well known that there is severe deficiency of *all* microions. This may result in a solvating medium so poor that under all conditions the mucus macromolecules are in a "gelled" state characteristic of any random coiling macromolecule, natural or synthetic, in a solvent below the so-called Flory theta temperature (37).

The difficulties encountered in obtaining the mucus of the normal respiratory tract in man are great. Since saliva is difficult to avoid as an artifact during collection, proper studies would require collection via direct aspiration of the bronchi. The important studies of Blanshard (43) on the mucus in various diseases of the lungs demonstrate the complexities of a few of the macromolecular substances produced in disease. In essence, therefore, the rheological study of bronchial mucus will be most applicable to material obtained from patients with lung disease, as in the case of cystic fibrosis wherein the retention of bronchial secretions, and the infection that accompanies these, are often the cause of death. There is adequate justification for careful study of the rheology of this mucus. The most striking example of needed data from a well-controlled study is that on the influence of iodides upon mucus viscosity. The extremely common use of saturated iodide solutions (e.g. potassium iodide) in the treatment of chronic bronchitis and similar conditions stands in bold contrast to the virtual absence of any critical study of the efficacy of these preparations.

These comments upon the rheology of three representative mucus fluids are obviously not designed to encompass the matter in encyclopedic fashion, but rather to emphasize two simple points: 1) the rheology of mucus from the standpoint of the engineer or physician justifies careful study based on sound principles of engineering and fluid mechanics, and 2) there is a relative paucity of information about mucus rheology that would appear to be quite correctable by present analytical techniques.

CONCLUSION

This review has briefly discussed the rheological properties of blood, blood plasma and certain mucus fluids. Their biologic functions are shown to be intimately related to their rheological behavior, principally as pseudo-plastic or shear thinning fluids. The efficiency or economy of the physiologic processes in man appear to take careful advantage of these phenomena.

This survey of whole blood, of blood plasma, and of mucus, as examples of biologically important fluids whose rheology is physiologically critical, has demonstrated to the reviewers that the systems of nature are far more complicated and specific in their actions than man-engineered systems. One is continually impressed by the intricate measures and countermeasures brought into play to meet different physiological situations. The spirit of nature's engineering is quite well conveyed in the article by Waugh (2) on blood clotting. The chain of interactions between separate mechanisms (for example, the triggering of blood clotting by leakage of juice from damaged tissue into adjacent vessels) places impressive barriers against even sampling these fluids from the living system, not to mention their analysis.

This review has not explored the equally important fluids of the synovial membranes (joint fluid), lymph, or the rheology of muscle. Each of these justify comprehensive discussion of themselves, for their rheological properties are no less critical than the fluids touched upon in this review.

There is no other approach to these matters than one of patience and humility. On the other hand, for those who can contribute to advances in these areas, it would seem to be unlikely that they would find problems more challenging or more worth doing than these. It is gratifying to note the trend of increasing and effective collaboration between engineers, physicians, natural scientists and physiologists on medically important problems, of which the subject treated in this review is but one example.

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Analytical Methods in Applied Mechanics

(See also Revs. 4574, 4576, 4588, 4599, 4601, 4602, 4615, 4621, 4663, 4677, 4695, 4700, 4740, 4746, 4773, 4795, 4822, 4842, 4885, 4887, 4939, 4969, 4974, 4980, 5071, 5084, 5086)

4567. Jones, J. G., On the numerical solution of convolution integral equations and systems of such equations, *Math. Comput.* **15**, 74, 131–142, Apr. 1961.

Consider the equation $bg(x) - \int_0^x u(x-t)g(t)dt = f(x)$ where b is a constant, $u(x)$ and $f(x)$ are known functions and $g(x)$ is to be found. The cases $a = 0$ and $a \neq 0$, equations of the first and second kinds respectively, are discussed. To solve the above equation, the interval $(0, x)$ is split into n equally spaced parts and the integral is approximated by the trapezoidal rule. This leads to a triangular system of linear equations from which approximate values of $g(x)$ can be determined at the mesh points. Considerable attention is devoted to the convergence question and, for both kinds of equations, it is shown that for x fixed, the error tends to zero as the mesh length goes to zero.

If $a = 0$, the equation may be solved directly or it may be put into the form of an equation of the second kind by differentiation. Author makes a comparison of the two methods with regard to propagation of rounding errors and illustrates the discussion with an example.

Finally, the analysis is extended to a system of equations of the above type where b is a constant square matrix, w is a variable square matrix, each of order m, and f and g are variable vectors in x, each vector having m components.

Y. L. Luke, USA

4568. Pope, D. A., A method of "alternating corrections" for the numerical solution of two-point boundary value problems, *Math. Comput.* **14**, 72, 354–361, Oct. 1960.

Consider the boundary-value problem $y'' = f(x, y)$, $y(0) = a$, $y(1) = b$. Partition the interval $(0, 1)$ into $(n + 1)$ equally spaced points and let y_j be an approximate solution at these points. Now partition $(0, 1)$ into $2n$ equal parts. Call the $(2n + 1)$ points x_j, $j = 0, 1, \ldots 2n$. Thus points of the original partition are given by x_j with j even. Approximate values of y_j for j odd are found using the interpolation formula

$$y_j = K(y_j) = \frac{1}{2}(y_{j+1} + y_{j-1}) - \frac{1}{4}h^2(y''_{j+1} + y''_{j-1}). \qquad (*)$$

This is called step A.

In step B, the values of y_j with odd and even index are corrected alternately using the same formula (*). If y^s_j is the value of y_j at the s-th iteration step, then as $s \longrightarrow \infty$, it is shown that the iteration process converges to the solution of the difference equations $y_j = K(y_j)$. Further, the solution converges to the solution of the given differential equation. An estimate of the rate of convergence is provided. Conditions for stability are derived. Two experiments are recorded. The results show good agreement with the theory.

Y. L. Luke, USA

4569. Tauber, S., and Dean, D., On difference equations containing step and delta functions, *J. Soc. Indust. Appl. Math.* **8**, 1, 174–180, Mar. 1960.

Paper first considers the effect of applying the forward difference and displacement operators on the Kronecker and Dirac delta functions. These results are then used to find particular solutions of linear difference equations with constant coefficients whose nonhomogenous parts are the above delta functions. Discussion has application to structural load problems.

Y. L. Luke, USA

4570. Kreiss, H.-O., Finite difference methods for the approximate solution of mixed problems for partial differential equations (in German), Trans. Roy. Inst. Tech., Stockholm no. 166 (Pure Appl. Math. Phys. no. 12), 61 pp., 1960.

Author considers finite difference methods for approximating partial differential equations of quasi-parabolic type (1st-order derivatives in t and mth order in x). General methods are given for constructing strongly stable difference approximations and necessary and sufficient conditions for their existence are given.

This is essentially a paper for pure mathematicians and the results as presented are not in a form useful to engineers or others who may need to use these methods.

J. M. Jackson, Scotland

4571. Zemanian, A. H., Further properties of certain classes of transfer functions, *Quart. Appl. Math.* **18,** 3, 223–228, Oct. 1960.

Some previously defined classes of rational transfer functions are extended to transfer functions that need not be rational. Several additional properties of these types of functions are developed. Bounds on certain derivatives of the unit impulse responses corresponding to such functions are shown to exist.

From author's summary by P. G. Bhuta, USA

Computing Methods and Computers

(See also Revs. 4567, 4601, 4602, 4672, 4799, 4801, 4876, 4923, 5092, 5128, 5139, 5168)

4572. Kincaid, W. M., A two-point method for the numerical solution of systems of simultaneous equations, *Quart. Appl. Math.* **18,** 4, 313–324, Jan. 1961.

Author presents a method which can be seen as a generalization of the *regula falsi* to the case of n equations for n unknowns. This iterative method does not require the calculation of derivatives or the solution of sets of linear equations like Newton's method but yet it displays second-order convergence. The solution is obtained by aid of $n + 1$ points which under certain conditions converge toward the required root. A proof of convergence is presented for the case $n = 2$. The conditions which are assumed for this proof are

(1) the three points chosen initially should lie within a certain neighborhood of the root;
(2) the three lines connecting the three initial points should not deviate too much from certain given directions.

However, it may be expected that convergence occurs under much wider conditions than those assumed in the proof. An illustrative example is added.

A. I. van de Vooren, Holland

4573. Karmishin, A. V., Method of solution of systems of algebraic equations of five terms associated with certain problems in structural mechanics (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **25,** 111–121, 1959.

A method of solution of a system of n linear algebraic equations is given, each consisting of at most five terms on the left side such that in the matrix of the system only the main diagonal and the two rows parallel to it on each side differ from zero. A numerical example is given for a system of 10 equations.

The method is to be used in structural mechanics for the solution of 3-, 4- and 5-moment equations and the approximate solution of linear differential equations of fourth order by solving the difference equations. In particular the method may be used to solve equations of the type

$$\alpha(x)y^{(IV)} + \beta(x)y^{(II)} + \gamma(x)y = F(x)$$

which occur in structural mechanics.

Edda U. Reeh, Germany

4574. Fehlberg, E., A new exact Runge-Kutta formula for n-th order differential equations (in German), *ZAMM* **40,** 10/11, 449–455, Oct./Nov. 1960.

Author extends his previous work [*ZAMM* **40,** pp. 252–259, 1960; AMR **14,**(1961), Rev. 2327] for second-order differential equations to the case of nth-order differential equations. More exact Runge-Kutta formulas are obtained by means of m-fold differentiation and suitable transformations. The formulas yield correct values, up to the h^{m+5} term inclusively, of the Taylor series for the solution and its derivatives. An advantage of these formulas is that they can be used to reduce machine time on electronic computers, maintaining the same accuracy as the standard Runge-Kutta formulas.

P. F. Byrd, USA

4575. Rakhmatulin, Kh. A., and Levit, D. E., One method of numerical integration of the equations of elastic equilibrium (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **28,** 224–240, 1960.

Authors consider the plane problem of elasticity with the help of the associated difference equation. They start with a lattice giving a set of algebraic equations that can be handled by a high-speed computer available to them. Then they consider a finer lattice. The resultant set of equations is solved by an interative method, the previous solution furnishing the initial values. A plane problem with mixed boundary conditions is numerically discussed.

P. Wilde, Poland

4576. Heller, J., Simultaneous, successive and alternating direction iteration schemes, *J. Soc. Indust. Appl. Math.* **8,** 1, 150–173, Mar. 1960.

The various iteration schemes for the solution of $(2p)^{th}$ order elliptic difference equations in two dimensions are studied from the point of view of minimizing a quadratic form and then from the point of view of separation of coordinates.

Because very little can be said about the convergence rates of iteration schemes of general elliptic difference equations of high order, author considers those difference equations in two dimensions in which the coordinates can be separated. From this point of view he studies, for $(2p)^{th}$ order elliptic difference equations, the convergence rates of line simultaneous, line successive and alternating iteration schemes. In particular he derives the eigenvalues of the error matrix for each of these schemes for any order of elliptic difference equations such that the coordinates are separable.

From author's summary by R. Conti, Italy

Kinematics, Rigid Dynamics and Oscillations

(See also Revs. 4601, 4602, 4603, 4700, 4701, 4705, 4714, 4802 5071, 5074, 5075, 5081, 5084, 5086, 5089, 5135, 5166)

Book—4577. Hinkle, R. T., Kinematics of machines, 2nd ed., Englewood Cliffs, N. J., Prentice-Hall, Inc., 1960, x + 353 pp. $7.50.

Book has the conventional coverage of motion, velocity, and acceleration; also the uses of complex variable methods. Intermediate and advanced subjects include dimensional synthesis, mechanical computing mechanisms and constrained motion, number synthesis. The book is suitable for beginning and advanced study.

C. R. Freberg, USA

4578. Marguerre, K., With respect to Lagrange's equation of kinetics and elastostatics (in German), *Ing.-Arch.* **28,** 199–207, Mar. 1959.

4579. Kuzmak, G. E., The motion of an axially symmetric solid in space about a stationary point under the action of moments slowly varying with time, *Soviet Phys.-Doklady* **5,** 3, 526-529, Nov./Dec. 1960. (Translation of *Dokladt Akad. Nauk SSSR (N. S.)* **132,** 3, 549-552, May 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4580. Ferrarese, G., Angular velocity in a rigid body motion and the local rotation of finite deformation (in Italian), *R. C. Mat. Appl., Univ. Roma* **18,** 1/2, 169-177, June/July 1959.

4581. Kondurar', V. T., Some questions arising from the problem of two bodies (in Russian), *Sb. Nauchn. Trudi Ivanovsk. (Energ.) In-ta* no. 8, 452-479, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11155.

The general problem is investigated regarding the progressive rotational motion of two absolutely solid bodies the elementary particles of which are being mutually attracted in accordance with Newton's law. A problem of this nature but dealing with the case of an arbitrary number of bodies was examined in a study by the abstractor [*Astron. Zh.* **35,** 2, 265-276, 1958] though in the coordinate form. The author investigates the problem of two bodies by applying the vectorial recording for the equations of motion. In the current work the equations for the motion are recorded in different systems for computations, classical integrals are derived and some of the simplest of the particular cases are discussed.

G. N. Duboshin
Courtesy Referativnyi Zhurnal, USSR

4582. Roitenberg, Ia. N., On the placing of a gyroscopic compass into a meridian during the acceleration of the rotors of the gyroscopes, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24,** 1, 113-119, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Paper presents a mathematical solution of the following problem: the rotor of a gyro-compass is given a specified acceleration during an interval $0 < t < T$. What are the values of the constant external couples that must be applied during three consecutive subintervals of the period $0 < t < T$ in order that the axis of the rotor should precess into a meridian at $t = T$? A numerical example is appended.

L. Maunder, Scotland

4583. Duffin, R. J., The Rayleigh-Ritz method for dissipative or gyroscopic systems, *Quart. Appl. Math.* **18,** 3, 215-221, Oct. 1960.

Author considers matrix equations of the type $Aq'' + Bq' + Cq = 0$ with different kinds of matrices B. Let B be a symmetrical matrix, then the system is called "overdamped" if, for the quadratic forms a, b, c of A, B, C, the condition $b^2 - 4ac > 0$ holds. The general solution of the equation of motion can then be written as sum of exponential decays. The decay values correspond in a certain way to the natural frequencies of conservative systems with $B = 0$. Therefore a "Rayleigh-Principle", which means that the natural frequencies of a conservative system cannot be decreased by imposing additional constraints to the system, can be formulated for the decay values too. It is shown that by means of a simple transformation the equations for a conservative gyroscopic system with the unsymmetrical matrix B can be converted into the equation of an overdamped dissipative system. In this way it can be shown that a Rayleigh-Principle is valid for conservative gyroscopic systems too.

K. Magnus, Germany

4584. Sobolev, S. L., Motion of a symmetric top with a cavity filled with fluid (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **3,** 20-55, 1960.

It is impossible to point out, in our brief report, all the beauty and exceptional mathematical elegance of this brilliant paper. It proves at the first glance a masterly knowledge of the finest questions of functional analysis as well as a perfect understanding of advanced technical and physical problems in mechanics and mathematical physics. It is an easy matter to comprehend this considering that the author is one of the leading authorities in many branches of mathematics and mathematical physics. Originality and a number of new results cannot be fully appreciated without a profound study of the paper. Of course, this presupposes a good familiarity with advanced facts of mechanics as well as an adequate knowledge of the theory of abstract spaces and of partial differential equations.

The subject is divided into three chapters. The first of them brings an excellent theory of a symmetric top with a cavity containing a perfect fluid; then comes the special case of an ellipsoidal cavity; and the last chapter is devoted to cylindrical cavities. It is the aim of the paper to determine, in any case, the stability conditions of the motion of the top.

Among a number of interesting results should be mentioned the complicated situation with which one has to deal in the case of cylindrical cavities. Here, by varying a characteristic parameter, we can obtain an infinity of domains where the motion of the top becomes an unstable one. In contrast to this, in the case of an ellipsoidal filling the intervals of stable and unstable motion depend in quite a simple way upon the roots of a cubical equation.

V. Vodicka, Czechoslovakia

4585. Ripianu, A., Motion of a system with a central force (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mashinostroenie* no. 6, 105-108, Nov./Dec. 1960.

A particle M is moving in the plane of three given points O, M_1, M_2 under the action of a central force pointing steadily to O. Is it possible to find such a trajectory of the moving particle that the center of the circle passing through M, M_1, M_2 would move with a constant velocity along the straight line which is perpendicular to the segment M_1 M_2 and passes through its center?

Evidently, one has here to do with an interesting generalization of Newton's classical considerations on parabolic motions. Mathematical formulation of the problem leads, however, to a complicated differential equation which cannot be solved in any closed form. The only favorable case is that where one of the given points M_1, M_2 coincides with the pole O. Then we come to a solvable Riccati equation and the trajectory appears in form of a complicated third-order curve.

The paper is a valuable contribution to analytical mechanics. It is of a kind suggesting a number of other interesting questions concerning the motion with a central force.

V. Vodicka, Czechoslovakia

4586. Baratta, Maria A., Effect of terrestrial rotation on the motion of a solid of variable mass (in Italian), *Mat. Parma* (2) **1,** 1, 21-27, 1960.

The movement equations of a solid with a variable mass under the influence of the earth's rotation are established, the weight and aerodynamic forces are considered negligible. It is also assumed that the solid maintains its gyroscopical structure during the whole movement. Under these conditions it is proved that there are movements of regular precession and even uniform rotations.

F. R. Marsicano, Argentina

4587. Popov, E. P., On the determination of higher harmonics on nonsymmetric auto-oscillations (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. Mash.* no. 6, 44-50, Nov./Dec. 1959.

Paper is a welcome improvement of two known methods for analyzing auto-oscillations of nonlinear systems. The first of them, N. M. Krylov's linearization of the problem, uses only the first harmonic from the development of the required solution into a Fourier series. Higher approximations are to be found with the aid of special asymptotic expansions. The second procedure operates with complete Fourier developments and is well-suited to the investigation of systems whose characteristics consist of several straight segments.

The method of the present paper takes into account any finite number of the beginning harmonics in the Fourier development of the wanted solution. Author starts with the result of the harmonic linearization which serves as a first approximation, which is successively improved by using higher harmonics of the required solution. A concrete example calculating with three harmonics illustrates this new method which leads to reliable results also in the case of nonlinear systems with much more general types of characteristics than those which can be treated by the two procedures mentioned at the beginning of this review.

V. Vodicka, Czechoslovakia

4588. Brunshtein, R. E., and Kobrinskii, A. E., Periodic motions of a system containing a small sphere in a cavity (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no 1, 10-21, Jan./Feb. 1959.

Considered is the motion of a simple two-mass system containing backlash which is acted upon by a harmonic force. Obtained and studied are motions accompanied by periodic collisions of both masses. It is shown that different periodic motions are possible for the same parameters of the system. The question of stability for these modes is investigated. Further, the system motion resulting from inelastic collision of masses is considered.

Stability diagrams for periodic motions are plotted. Some experimental verification is presented in the form of oscillograms showing a few periodic and aperiodic motions that are possible.

V. Chobotov, USA

4589. Kononenko, V. O., Forced vibrations of systems with a source of energy in the case of small nonlinearities (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 6, 80-85, Nov./Dec. 1960.

Paper examines the influence that the properties of the source of energy have on the stability of a vibrating system with a nonlinear elastic force. An engine serves as the energy source, the vibrations are excited by the inertia forces of an unbalanced mass which is connected with the rotor of the engine.

General deductions are based upon asymptotic methods since they are of such frequent use in the theory of nonlinear vibrations. A concrete example concerns the case of a cubically varying elastic force. It serves to emphasize and graphically illustrate some remarkable facts which are to be considered as valuable results in the field of nonlinear vibrations.

V. Vodicka, Czechoslovakia

4590. Osmakov, S. A., An approximate method for the determination of the mean velocity of the motion of a particle along a horizontal vibrating surface (in Russian), *Izv. Vyssh. Uchebn. Zavedenii, Str-vo i Arkhitekt.* no. 5, 94-100, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11191.

The influence of a regime of directed vibration of a horizontal surface on the mean velocity of the motion of a particle v over that surface is investigated. In order to simplify the solution an approximate method of analysis is chosen starting from the assumption that the motion of the particle over a cycle follows the scheme: a free flight, an inelastic momentary impact with the surface accompanied by a relative halt of the particle for a time duration of Δt. It is shown that this motion is not the only possible form for the periodic motion but should possess the highest degree of stability. A comparatively simple solution of differential equations for the motion of the particle is achieved by resolution into a series of functions $\cos(\omega t)$ (ω being the vibration's frequency, t the time) and by discarding the high powers of (ωt). It is indicated that v is the function of the velocity of the vibrations of the surface, α, ω, its acceleration $\alpha\omega^2$ and the angle of vibration. Graphs are furnished for the simplified quantitative calculations.

V. N. Geminov

Courtesy Referativnyi Zhurnal, USSR

4591. Gulyaev, M. P., The circular sections of reciprocal ellipsoids of inertia (in Russian), *Trudî Sektora Matem. i Mekhan. Akad. Nauk KazSSR* **1**, 175-193, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11184.

Some of the properties of circular sections of reciprocal ellipsoids of inertia are formulated: the conditions relating to conjugation and the orthogonal nature of circular sections which during the conversion of ellipsoids change over from one to the other; the normals of the circular sections which are asymptotes of the geometrical location of the spheroidal points; the planes of the circular sections for different poles lying on the normal to the circular section of the central ellipsoid of inertia passing through the center of gravity of the body which preserve identical orientation in space. The orientation is explained of the planes of the circular sections for poles situated on the principal axes of the central ellipsoid of inertia. There is a discussion on the question of the dynamic treatment of the reciprocal ellipsoids of inertia.

L. M. Markhashov

Courtesy Referativnyi Zhurnal, USSR

Instrumentation and Automatic Control

(See also Revs. 4582, 4899, 5026)

Book—4592. Zypkin, Ja. S., Theory of automatic relay control systems [Theorie der Relaissysteme der automatischen Regelung], Munchen, R. Oldenbourg, 1958, 472 pp.

The book is concemed with feedback control systems which include a relay as the only nonlinear component of the loop. For relays without a dead zone the output is constant and depends only on the polarity of the input signal. Symmetrical and unsymmetrical relays with and without dead zones and hysteresis are considered. Methods of analysis are developed which are analogous to those used for linear feedback control systems such as transfer functions, frequency characteristics and impulse response. A brief survey of the literature is followed by a description of automatic relay control systems and their classification. The equations of such systems are formulated in time and frequency domains as well as in the Laplace transform form. Numerical methods for the calculation of transients are given. Stability of equilibrium and steady-state periodic oscillations are investigated, and free and forced oscillations are considered. Approximation methods including linearization are presented and evaluated. Mathematical appendices on the Laplace transformation, Fourier series and the Dirac delta-function are included. Statistical methods and noise are not considered.

The book is intended for the reader who is familiar with the theory of linear feedback control systems. The clarity of the exposition is superb. Because of its completeness and readability this book is highly recommended to engineers.

C. Saltzer, USA

Book—4593. Oppelt, W., Short handbook of technical control processes [Kleines Handbuch technischer Regelvorgange], 3rd revised ed., Weinheim, Verlag Chemie, 1960, 632 pp. DM 39.60.

This third expanded edition of the well-known German introductory book on automatic control has now reached the size where it is not adequately called a "short" handbook. The expansion from 555 pages in the second edition [AMR **10**(1957), Rev. 1694] to 632 pages is mainly due to the addition of more examples and fields of application. Major new material is the chapter on digital control systems.

The author has intentionally avoided the use of more mathematics than necessary since the book is also intended as en elementary textbook. This does not, however, greatly affect the generality of the presentation because of the thorough treatment explained by illustrative diagrams in a clear and systematic way.

Besides this the book serves its purpose as a handbook by means of tables and standard curves. Emphasis is put mainly on the engineering aspect of control theory but without hardware details. The drawback in the first edition of using a terminology which differed from most international (in particular American) literature has now been mostly removed.

F. Y. Hjel·e, Sweden

4594. Mikhailov, N. N., The equation and certain properties of an automatic control system's root locus, *Automation and Remote Control* **20,** 8, 1063–1070, Apr. 1960. (Translation of *Avtomatika i Telemekhanika* **20,** 8, 1095–1102, Aug. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

By using the methods of the complex variable a complete topological discussion of the behavior of the poles and zeros of the characteristic equation of a linear control system when a system parameter is varied is given in this paper. The analysis is presented for the general case in which the characteristic function is the ratio of a polynomial of the mth degree and a denominator polynomial of the nth degree. The general theory is then applied to special cases.

L. A. Pipes, USA

4595. Yakab, I. G., Calculation of periodic regimes in linear systems (in Russian), *Naucbn. Dokladi' Vyssh. Sbkoly. Elektromekb. i Avtomatika* no. 2, 46–53, 1958; *Ref. Zb. Mekb.* no. 10, 1959, Rev. 11241.

The method of resolution of the extemal perturbation into a Fourier series is adopted for the calculations of a steady regime reacting on a linear system of periodic nonharmonic extemal forces. But this method not infrequently gives unwieldy and inconvenient expressions for the calculations. The present paper advances a method based on separating out the periodic portion from the general solution of the differential equation describing the process. This line of approach results in the obtaining of a more compact and more graphic expression which in structure agrees with the general methods of calculation for electrical circuits.

S. V. Kalinin

Courtesy Referativnyi Zhurnal, USSR

4596. Lakshmi Bai, Mrs. C., Composite describing function of successive non-linear components in feed back control systems, *J. Indian Inst. Sci.* **43,** 1, 52–64, Jan. 1961.

Recent work by the author [Trans. A.I.E.E. June 1960, Paper No. 60–865) on a transformation method for the study of nonlinear components is utilized to evaluate the composite describing function of successive nonlinear components in feedback control systems. Characteristic equations for different values of the nonlinearizing parameters are given for commonly encountered nonlinear components in control system practice. Composite describing functions are evaluated in case of a ba klash followed by a dead-zone, and also in case of a rate-limiting device followed by a variable gain amplifier. In the latter case, the composite describing function is dependent on the input amplitude and frequency.

From author's summary by R. D. Milne, England

4597. Emel'yanov, S. V., The use of nonlinear correcting devices of the "key" type for improving the quality of second-order automatic control systems, *Automation and Remote Control* **20,** 7, 844–859, Mar. 1960. (Translation of *Avtomatika i Telemekhanika USSR,* **20,** 7, 867–883, July 1959 by the Instrument Society of America, Pittsburgh 22, Pa.)

4598. Volgin, L. N., A method of synthesizing linear sampled-data automatic control systems to accord with dynamic criteria, *Automation and Remote Control* **20,** 10, 1319–1326, June 1960. (Translation of *Avtomatika i Telemekhanika USSR* **20,** 10, 1350–1356, Oct. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Paper presents standard procedure for synthesizing sampled-data feedback system by constraining closed-loop transfer function to be minimal polynomial in z multiplied by an "artificial" rational function introduced to give acceptable quality, e.g. minimum mean-square error. Paper is mainly expository. Note author defines z as $\exp(-sT)$ not $\exp(sT)$.

N. Ream, England

4599. Serebryakova, N. N., On the behavior of a dynamic system with one degree of freedom in the vicinity of those points of the boundary of the stability region at which "nondangerous" boundary passes into a "dangerous" one (in Russian), *Izv. Akad. Nauk SSSR Otd. Tekb. Nauk, Mekb. i Mash* no. 2, 178–182, 1959.

The work considers the behavior of a system near those points on the boundary of stability where the first Lyapunov quantity is zero. The behavior of the system near the boundary cannot in this case be classified as dangerous or nondangerous depending on the sign of the first Lyapunov quantity. The behavior can thus be characterized by the sign of the second Lyapunov quantity.

The paper computes the second Liapunov quantity in terms of the coefficients of a canonical system and studies the stability of the system near its boundary at points where the first Lyapunov quanity is zero.

V. Chobotov, USA

4600. Perel'man, I. I., Choice of an optimum amplification factor for a self-aligning control system, *Automation and Remote Control* **20,** 2, 177–184, Jan. 1960. (Translation of *Avtomatika i Telemekhanika* **20,** 2, 184–191, Feb. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

Author extends analysis of control system originally described in 1958 [AMR **13**(1960), Rev. 3824] to determine the loop gain which minimizes the r.m.s. error when the system is subject to intermittent or cyclic random disturbances with nonzero mean value. The control loop contains one integration, all other time-dependent effects being neglected. Application is suggested to the control of finished product thickness in hot-rolling strip mills.

G. D. S. Maclellan, Scotland

4601. Kirillova, F. M., A limiting process in the solution of an optimal control problem, *Appl. Math. Mech.* (*Prikl. Mat. Mekb.*) **24,** 2, 398–405, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Let

$$dx/dt = A(t)x + b(t)u(t);\ x(0) = x_0 \qquad [1]$$

where x is an n-dimensional vector. Here $A(t)$ is an $n \times n$ matrix and $b(t)$ an n-vector, continuous in t. Let the control function $u(t)$ satisfy the condition

$$\int_{t_0}^{t} |u(\tau)|^p \, d\tau \leq 1, \qquad (p > 1). \qquad [2]$$

The optimal problem under consideration is the following. Among all control functions satisfying [2] it is required to determine $u(\tau, p)$ such that the solution of [1] $x = x(t; x^0, u)$ reaches the origin in the shortest time. This time, $T(p)$ is called the *optimal time*; the corresponding control function $u(\tau, p)$ is called the *optimal control function*.

The interest in the condition [2] for arbitrary $p > 1$ stems from the possibility of going over to the condition

$$|u(\tau)| \leq 1 \qquad (t_0 \leq \tau \leq t) \qquad [3]$$

as $p \longrightarrow \infty$. Thus a method is provided for finding an approximate optimal solution under constraint [3]. One replaces it by constraint [2], yielding a problem which can be solved by the calculus of variations; then let $p \longrightarrow \infty$. H. D. Block, USA

4602. Krasovskii, N. N., Approximate computation of optimal control by the direct method, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24,** 2, 390–397, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The first problem considered is the following. Let $dx/dt = Ax + bu$, where x is an n-dimensional vector, A is an $n \times n$ matrix, b is an n-vector and $u(t)$ is a scalar function that describes the control action; $u(t)$ *is admissible* if $|u(t)| \leq 1$. Given that $x(0) = x_0$ it is required to determine an admissible function $u^0(t)$ (optimal control) such that the trajectory $x^0(t) = x(x_0, t, u^0)$ will arrive at $x = 0$ in the shortest possible time. An approximate method for computing $u^0(t)$ and $x^0(t)$ is given in terms of auxiliary problems which are reduced to a classical variational problem. This in turn is solved by a direct method. H. D. Block, USA

4603. Matveev, V. I., The double pendulum as a mechanical part of equipment required for the automatic control of the vibrations of the bearings of pit-shaft ventilators (in Russian), *Trudi Donetsk. Industr. In-ta* **26,** 119–126, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11303.

Equipment designed by the author for the automatic control of vibrations is described. The apparatus takes the form of a double pendulum suspended on a cross piece to the body. The edges on which the cross piece is suspended to the body are disposed perpendicularly to the axes of the suspension arm of the pendulums; this relieves the double pendulum of forces directed along the axes of the pendulum. When the body of the apparatus, which is clamped to the controlling member, is vibrating, the lower pendulum remains practically motionless, while the upper one vibrates. In consequence the pointer fastened to the upper pendulum turns relative to the lower pendulum. Should the vibrations exceed the normal the pointer makes contact on the lower pendulum and completes the circuit of the signaling device. The suspension arm described above makes the clearance between the pointer and the contact independent of the inclination of the body and ensures control of the vibrations up to 0.01 mm. The experimental characteristic brought in shows that the working zone of the apparatus, in which the characteristic is very nearly rectilinear, occupies the portion from 9.8 to 35 Hertz. The theoretical frequency characteristic is also brought in; this is obtained on the basis of the solution of the differential equations for the forced vibrations of the double pendulum with no account being taken of the resistance forces. The offshoots of the theoretical characteristic delimit the working zone of the apparatus with sufficient precision to allow the formulas obtained to be used when designing similar apparatus.

A. A. Gusarov
Courtesy Referativnyi Zhurnal, USSR

4604. Ceaglske, N. H., Automatic control of fluid flow, *AIChE J.* **5,** 4, 524–532, Dec. 1959.

4605. Gaylord, R. S., Differentiating gas jet for space attitude control, *ARS J.* **31,** 1, 75–76 (Tech. Notes), Jan. 1961.

4606. Cheprasov, V. P., The equations for the curvilinear motion of an elastic blade (in Russian), *Trudi Kazansk. Aviats. In-ta* **43,** 87–102, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11561.

A system is derived consisting of five differential equations for calculating the motion of a blade of a carrying screw in the presence of a swing regulator, horizontal and vertical hinges and automatic curvature control. It is assumed that the angular velocity of the turning of the screw is less than 0.1 of the angular velocity of rotation; the calculation of the aerodynamic loading is based on the hypothesis of the stationary state; the inductive velocity is found in the form of trigonometrical series.

G. I. Maikapar
Courtesy Referativnyi Zhurnal, USSR

Tables, Charts, Dictionaries, etc.

(See also Rev. 4812)

Book—4607. International dictionary of applied mathematics, Princeton, N. J., D. Van Nostrand Co., Inc., 1960, 1173 pp. $25.

In this comprehensive tome, terms and methods of application of mathematics to numerous fields of science are defined and explained. Concepts of pure mathematics necessary to an understanding of ideas in the applied fields are also covered. This broadens the usefulness of the book. The material has been assembled by a prominent group of 34 workers, mostly scientists who are well known in the fields of mathematics, physics, and engineering. Concepts are discussed in sufficient detail to permit the adaptation of methods from related fields. For example, electrical circuits are considered from the point of view of stability, topology, and dynamical analysis. A valuable feature is an alphabetical listing of scientific terms in French, German, Russian and Spanish, along with their English equivalents. Thus, given a scientific term in Russian, the language dictionary is used to find its English counterpart and definition of the latter is then found in the main part of the encyclopedia.

Y. L. Luke, USA

Book—4608. Ferraro, A., Dictionary on the theory of measurements [Piccolo dizionario metrologia generale], Bologna, Nicola Zanichelli Editore, 1959, x + 292 pp. 3.500 Lire.

Book contains, in alphabetical order, words with physical significance, their definition, symbol or abbreviation and, eventually, the corresponding dimensional expression. Author uses preferably Giorgi system, following the resolution of I. C. W. M., Paris 1946. In each case, equivalence with other systems in common use is given.

Reviewer points out the fact that Sevres' meter is mentioned as length standard. When book was published (1959) this was true, but in October 1960 a new, spectrographic standard was adopted; this is an unfortunate circumstance for the book. Reviewer could not confirm—as the corresponding article says, p. 125—that Celsius degree is the unit for temperature measurement in Giorgi system; in the careful article on "Giorgi," p. 119, temperature is not mentioned. In practice there is no preferred scale. Nevertheless it seems reasonable that the general validity of expression: energy = $\frac{1}{2} kT$ (by degree of freedom), requires the general use of Kelvin scale, despite the fact that degree's length is the same as in Celsius one.

Articles are short and clear; numerical tables complete them. Work as a whole is beautifully presented; reviewer believes it is a good addendum to everyone's library.

O. Cerceau, Venezuela

Elasticity

(See also Revs. 4632, 4654, 4663, 4671, 4687, 4717, 4719, 4734, 4741, 4789, 4794, 5130)

4609. Slesinger, I. N., On the variational theorems of the nonlinear theory of elasticity (in Russian), *Buletinul Institutului Politebnic Din Iasi* **5 (9)**, 3/4, 83–86, 1959.

In this paper it is shown that the variational theorems of Novozhilov, Reissner, and Galimov of the nonlinear, and of Castigliano of the linear, theories of elasticity are particular cases which can be derived from the general variational principle given by the functional

$$J/u_m, K_{ij}, \kappa_{ij}/ = \iiint_V \{\Phi(\kappa_{ij}) - K_{ij}\kappa_{ij} - g_{ij}(\lambda_{mn})\} - F_m u_m\} \, dV -$$

$$\iint_{S'_m} f_m u_m \, dS - \iint_{S''_m} K_{ij} \varphi_{ijmn} \beta_n (u_m - v_m) \, dS,$$

$$\varphi_{ijmn} = \partial g_{ij}/\partial \lambda_{mn}, \quad \lambda_{mn} = \partial u_m/\partial x_n, \qquad (i,j;\ m,n = 1, 2, 3)$$

Notations used:

u_m—components of displacement in directions of orthogonal cartesian axes x_m

K_{ij}—generalized stresses corresponding to the adopted characteristics of deformations κ_{ij}

V—volume of the body before deformation

S—boundary surface of the considered body

Φ and F_m—potential energy and body forces per unit of V

f_m—surface forces per unit of S

g_{ij}—given functions of λ_{mn}

S'_m—parts of S where f_m are given

S''_m—parts of S where components of displacement are given

β_n—direction cosines of external normal to S.

V. Kovarik, Czechoslovakia

4610. Sinitskii, A. K., Linkage between stresses and deformations in a nonlinear elastic continuous medium (in Russian), *Trudi Leningr. Politekbn. In-ta* no. 196, 129–143, 1958; *Ref. Zh. Mekb.* no. 10, 1959, Rev. 12190.

Relations are derived connecting the stresses and the deformations while taking into account possible divergences of the deviators of the stresses and deformations from the principle of similarity. The following expressions for the principal components of the deformation deviator are given

$$\varepsilon_3 - \varepsilon_m = \frac{\gamma_m}{\sqrt{2}} \sin \eta$$

$$\varepsilon_3 - \varepsilon_m = \frac{\gamma}{\sqrt{2}} \sin \left(\eta - \frac{4}{3}\pi\right)$$

where

$$\varepsilon_m = \frac{\varepsilon_1 + \varepsilon_2 + \varepsilon_3}{3}$$

and

$$\gamma_m = \frac{2}{3}\sqrt{(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 + \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$$

If attention is given to the fact that $\eta = \beta_\varepsilon - \pi/6$ and $\gamma = \sqrt{2}\,\varepsilon_i$ it will be found that the above expressions lead to the expressions known in the literature. The author arrives at an expression corresponding to the principal components of the deformation deviator by using geometrical constructions. The author utilizes the parities derived to give a more symmetrical form to the recorded series of relations for the nonlinear theory of elasticity.

G. A. Smirnov-Alyaev

Courtesy Referativnyi Zhurnal, USSR

4611. Livshits, P. Z., State of stress in elastic cylinders, loaded on the lateral surface by tangential forces (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **30**, 47–56, 1960.

Paper deals with the elastic equilibrium of an infinite circular cylinder, loaded in axial direction by two equal and opposite forces, uniformly distributed over two equal annular strips of the lateral surface. Expressing the external forces by the use of Fourier integral, author gives a solution based on Papkovich-Neuber method [*Izv. Akad. Nauk SSSR Phys.-Math. Ser.*, pp. 1425–1435, 1932; *ZAMM* **14**, p. 203, 1934] which has been further developed by A. I. Lur'e [AMR **9**(1956), Rev. 2850].

C. Hoschl, Czechoslovakia

4612. Silkin, E. I., Analysis of a semi-phane weakened by a circular annulus (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **30**, 57–65, 1960.

Author considers a semiplane uniformly loaded on its edge and weakened by a circular annulus, which is assumed to be at a sufficient distance from the edge and has different elastic properties. Using Airy's stress function, author gives a solution for stresses and displacements in the neighborhood of the annulus. The contact surface of the annulus is assumed to be either smooth or rough. A numerical example is given.

C. Hoschl, Czechoslovakia

4613. Aleksandrov, V. M., and Vorovich, I. I., The action of a die on an elastic layer of finite thickness, *Appl. Math. Mech.* (*Prikl. Mat. Mekb.*) **24**, 2, 462–476, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The problem is solved by using the expressions of normal displacements to the layer and those of stresses under the form given by Lur'e ["Three-dimensional problem of the theory of elasticity," GITTL 1955; AMR **9**(1956), Rev. 2850] which were obtained with the aid of Fourier transformations, and with which the boundary conditions are satisfied. The condition obtained is an integral equation of the first kind

$$\iint_\Omega \frac{2(\xi,\eta)\, d\xi d\eta}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} = 2\pi\Delta\delta(x,y) - \frac{1}{\pi} \iint_\Omega L(\xi,\eta)(K - K_\infty)\xi d\eta$$

L being the load on the upper boundary and δ the constant part of the displacement w. Zero tangential stresses and $w = 0$ are assumed on the lower boundary.

By developing the nucleus $K - K_\infty$ into power series, the mentioned equation is split into a series of recurrent integral equations where the unknowns appear also under the integral sign and have the same nucleus.

For the case of a flat elliptic die, elliptical coordinates are used in the manner mentioned by L. A. Galin ["Contact problems of the theory of elasticity," GITTL, 1955].

The solution q is finally expressed under the form of a polynomial in a/b (a is the semi-axis, b is the layer thickness) whose coefficients are listed in tables.

The following corrections are to be made in the text: $2(m-1)$ should be considered instead of $2(m-2)$, $\varepsilon^{-2\gamma b}$ should be read instead of $f^{-2\gamma b}$ and lz^2 should be taken instead of l^2x^2.

M. M. Misicu, Roumania

4614. Tarabasov, N. D., State of stress in a multiply connected semi-plane (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **25**, 136–144, 1959.

Author applied the method of singular integral equations to determine the state of stress in a half-plane punctured by circular holes each of which is filled by material, with forces and couples applied to the filler, the problem of the riveted plate. An explicit solution is obtained and formulas for the stresses are given.

L. M. Milne-Thomson, USA

4615. Rudiger, D., A generalization of the principle of minimum potential energy for two-dimensional elastic continua (in German), *Ing.-Arch.* **29**, 2, 115–124, Apr. 1960.

A variation principle of two-dimensional elastic continua is presented which contains the principle of the minimum of potential energy and Castigliano's principle as special cases. Since there are no boundary conditions of the variational principle, the functions used in the procedure by Ritz and Trefftz can assume arbitrary values at the boundary.

From author's summary by C. B. Ludwig, USA

4616. Muki, R., and Sternberg, E., Steady-state thermal stresses in an elastic cone (in English), *ZAMP* **11**, 6, 471–496, Nov. 1960.

The thermoelastic problem investigated considers the thermal stresses produced in a solid, homogeneous, isotropic, elastic cone of semi-infinite extent free of external loads and subject to a stationary discontinuous surface temperature. The tip of the cone up to a given distance from the apex is at T, while the remainder of the boundary is maintained at T_z. The heat conduction solution was presented in a previous paper by the authors and therefore they proceed directly to the thermal stress problem by a modification of the axisymmetric version of Papkovich's general solution of the elastostatic field equations. By use of the Mellin transform and corresponding inversion theorem, suitable integral representations of the thermal stresses are deduced. Numerical results were obtained for a 60° cone and are presented in graphical form.

G. Gerard, USA

4617. Stanisic, M. M., and McKinley, R. M., On the steady thermoelastic problems with elastic characteristics depending on temperature (in English), *Ing.-Arch.* **29**, 4, 260–261, Aug. 1960.

A comparison is made between a method developed by Trostel [AMR **11**(1958), Rev. 4882; and *Ing.-Arch.* **26**, 416–434, Dec. 1958] and an extension of this method by the authors [AMR **14**(1961), Rev. 1230]. The methods were developed for determining the elastic thermal stresses in a body in which the coefficient of expansion and modulus of elasticity are a function of temperature. Trostel's solution is a series of solutions having a "basic solution" plus "correction solutions." He evaluates the solution for an axisymmetric problem using the "basic solution" and one "correction solution" term.

Present authors' solution for one-dimensional axisymmetric problems replaces the compatability equation by an approximation which becomes exact when Poisson's ratio is 1/2. The approximate equations are then solved by quadratures.

In this note the authors compare the two solutions. They show that the approximate method developed by them is equivalent to the "basic solution" of Trostel plus part of his first "correction solution," neglecting, of course, any errors introduced by the numerical integration. The authors state that the neglected portion of the "correction" constitutes less than 30 per cent of this "first correction term."

The authors' evaluation of this accuracy appears to be based on the numerical solutions using a value for Poisson's ratio of 1/4.

A. L. Ross, USA

4618. Trommel, E., Thermal stresses during hardening of concrete (in German), *Öst. Ing.-Arch.* **13**, 2, 58–81, 1959.

Author discusses the problem of circular cylinders of constant length and rectangular prisms of infinite length made of a homogeneous, isotropic and elastic material and subjected to unsteady temperature field. It is considered that the temperature field is due to arbitrarily distributed heat sources whose intensity varies exponentially in time. The uncoupled equation of heat propagation is used and a particular integral is sought for the case when the intensity of heat sources is given under the form of double series with respect to the space variables; to this integral, the general integral of the homogeneous equation is added under a similar form. The results obtained are examined for concrete bars during hardening.

In the second part of the paper the strain and stress state is determined proceeding from the Duhamel-Neumann equations and using the method given by Melan and Parkers. The possibility of studying the subject when the elastic constants of the material vary exponentially in time is briefly examined.

P. P. Teodorescu, Roumania

4619. Manokhin, D. G., Thermal stresses in thin-walled constructions (in Russian), *Trudi Kazansk. Aviats. In-ta* **43**, 43–53, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12446.

A multilinked cylindrical beam fitted with sheathing fixed onto the lattice skeleton consisting of longitudinal and transverse ribs is examined. It is assumed that the transverse section is not subject to distortion and that the transverse ribs out of their own plane possess small rigidity. The temperature of the ribs and the sheathing is constant with the thickness and only varies over the transverse section and along the generatrix. The modulus of elasticity and the coefficient of expansion are assumed to be constant. A differential equation is derived and solved for the deplanation as laid down by Yu. G. Odinokov [*Trudi Kazansk. Aviats. In-ta* no. 18, (1946); no. 23 (1948)]. Solutions are given for two numerical examples when four of the twelve ribs of the skeleton were heated from 20 to 120°. When the temperature in the transverse section was distributed according to the plane principle no stresses were produced.

N. K. Snitko

Courtesy Referativnyi Zhurnal, USSR

4620. Zakharov, K. V., The problem of the approximate solution of the deflection of an orthotropic strip (in Russian), *Nauchno-Tekhn. Inform. Byul. Leningr. Politekhn. In-ta* no. 1/2, 187–192, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12255.

An approximate solution is given [See K. V. Zakharov, Trudi Leningr. Politekhn. In-ta, no. 192, 1957] for the special case of an orthotropic strip of rectangular section, rigidly clamped at one end and loaded by an evenly distributed transverse load along its length. An investigation is made of the influence of the nonlinearity of the change of the deflection moment along the length of the beam on the accuracy of the solution. A comparison with the precise solution [See S. G. Lekhnitskii, "Anisotropic plates," Moscow, Gostekhteorizdat, 1947] established the fact that the magnitude of divergence for normal stresses reaches 6%. This is twice as large by comparison with differences obtained in problems in which the deflecting moment varied linearly. The magnitude of divergencies for tangential stresses was very insignificant. The transpositions calculated by the approximate solution differ by 4 to 8% from the transpositions obtained by the formulas of the precise solution of the problem. When the linear principle of change of the deflection moment is in force the magnitude of divergencies was 3 to 6%.

N. G. Kushelev

Courtesy Referativnyi Zhurnal, USSR

4621. Abramian, B. L., and Babloian, A. A., Torsion of circular rods having longitudinal notches or teeth and a central cavity, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24**, 2, 487–499, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

The torsional stress function of hollow uniform cylinders with annular cross section comprised of (1) inner smooth circle and

outer uniformly notched circle, or (2) outer smooth circle and inner grooved circle is effected by considering the cross sections as composed of a cyclic repetition of symmetrical regions each of which is, in turn, made up of three curtate sectors. The torsional stress function is expressed as an infinite series of hyperbolic trigonometric functions within each of the three regions, with constant coefficients so determined through solution of infinite sets of algebraic equations as to satisfy the appropriate boundary conditions on the segments of the boundary circles and to match at the common interfaces of the curtate sectors.

Numerical values of the torsional stiffness are determined for the three cases of six external notches and smooth inner perimeter, of one external notch and smooth inner perimeter, and of smooth outer surface and six inner notches, of numerically specified dimensions. Finally, as a check on the general correctness of the analysis and for comparison with the just-mentioned cases, the torsional stiffness is determined for several numerically specified cylinders of circular annular cross section, whereof the exact solution is well-known.

The paper ought be of interest to all concerned with the theory of elasticity. It exhibits a clever and unique mode of solution, pioneered in USSR papers. The particularly interested reader can find a good account of the general procedure in the recently translated text by Kantorovitch and Kryloff, "Methods of approximate analysis" [AMR **12**(1959), Rev. 4830].

T. J. Higgins, USA

4622. Abbassi, M. M., Stresses in curved beams of noncircular center line, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 445–454, Sept. 1960.

Paper deals with general case of a curved beam with plane, noncircular center line. Three-dimensional equilibrium, compatibility and boundary condition equations, all in terms of stress, are derived in suitable orthogonal curvilinear coordinates. Problems solved include cantilevers with elliptical and circular cross sections, in bending (out of plane) and torsion; and also under concentrated end load, perpendicular to the plane of the beam. Solution is by successive approximations (following Gõhner), starting from Bernouilli-Euler bending and Saint-Venant torsion for straight beams as the first approximation. Second approximation is obtained by neglecting terms of order $(\xi/\rho)^2$ and higher in the equilibrium equations, and ξ/ρ and higher in the compatibility equations. Here ρ is the local radius of curvature of the beam, and ξ represents a linear dimension of the cross section. Results are thus valid for slender curved beams.

J. L. Lubkin, USA

4623. Kalandiya, A. I., A plane problem of the Hertz type on the compression of cylindrical bodies (in Russian), *Soobshch. Akad. Nauk GruzSSR* **21**, 1, 3–10, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12214.

An integral equation is derived by the N. I. Muskhelishvili method for the plane problem on the stresses in an infinite elastic medium with a round orifice into which has been set an elastic collar made from a different material and having initially a somewhat smaller radius. For this equation, which coincides with Prandtl's equation in the theory of a wing of finite span, a different deduction is given in I. Ya. Shtaerman's "Kontaktnaya zadacha teorii uprugosti" ["A contact problem in the theory of elasticity," Moscow-Leningrad, 1949]. Numerical methods for the solution of this equation are given in the author's paper [*Prikl. Mat. Mekh.* **21**, 3, 389–398, 1957] and in the monograph referred to above.

N. A. Rostovtsev

Courtesy Referativnyi Zhurnal, USSR

Viscoelasticity

(See also Revs. 4677, 4725, 4835)

4624. Likhtman, V. I., and Shchukin, E. D., Physicochemical phenomena associated with the deformation of metals (in Russian), *Uspekhi Fiz. Nauk* **66**, 2, 213–245, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12655.

Paper is a review of work done to establish the influence of surface-active molten solutions of tin, lead and mercury on the deformation and destruction of monocrystals of zinc, tin and cadmium. The character of the influence of the surface-active medium depends on the temperature and the other conditions of the experiment. For instance, the tin melt sharply lowers the level of the normal stresses of the break-away of zinc in conditions of tension with a significant lowering of the plasticity (the magnitude of the deformation up to disruption) and strength. However, in conditions of creep when the constant stress acting on the metal is lower than the stress resulting in brittle break-away, in the presence of a surface-active melt an adsorption effect of plastification makes its appearance; it manifests itself by a marked increase in the velocity of the creep. If the influence of brittleness on tin shows itself from 350 to 400°, then, under the action of mercury, monocrystals of zinc already pass over to the brittle state at room temperature. The authors confirm the non-observance of the principle of critical normal stresses for the destruction of monocrystals of zinc [*see:* A. Deruyttere, G. B. Greenough, *J. Inst. Metals* **84**, no. 9, 337–345, 1955–56]. The theoretical line of argument is based on known presentations regarding the clogging of dislocations lying in the general plane of slip [A. N. Stroh, Advances in Phys., **6**, 24, 418–465, 1957]. The transition from plasticity to brittleness corresponds to the change of probability regarding the formation of dangerous aggregations of dislocations due to the increase in the micro-nonhomogeinity of the deformations and the minimizing of the part played by the resorption of the dislocations, stopped by the obstacles. The nature of the obstacles is not given a concrete form. The influence of the surface-active substances is determined by the migration of their atoms toward the defects of the structure and the lowering of the energy of the new surfaces of the division, which facilitates the development of cracks. No account is taken of the fact that the model adopted by the authors does not give normal stresses, acting on the plane of slip at the time when the monocrystals being investigated are in possession of the capacity of cleavage on the base, that is by the surfaces of slip [*see:* V. N. Rozhanskii, *Doklady Akad. Nauk SSSR* **123**, 4, 648–651, 1958; A. N. Stroh, *Phil. Mag.* **8**, 30, 597–606, 1958].

V. L. Indenbom

Courtesy Referativnyi Zhurnal, USSR

4625. Mendelson, A., and Manson, S. S., The extrapolation of families of curves by recurrence relations, with application to creep-rupture data, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 839–847, Dec. 1960.

The method presented in this paper enables one to extrapolate directly a family of curves such as creep-rupture curves without assuming any particular parametric form. The method is based on the assumption that a family of curves such as a set of isothermal curves can be represented approximately by a finite-difference recurrence relation. Once the coefficients of this recurrence relation are determined, it is a simple matter to extrapolate each member of the family individually. The method has the advantage that it does not require an explicit knowledge or assumption as to the analytic character of the curves or as to what parametric form to use to correlate the data, although implicity it assumes any of the broad class of functions which satisfy linear finite-difference equations. It is thus much more general in nature than the

parametric methods previously discussed and can be used by itself to extrapolate creep-rupture data or as an independent check on any of the parameter methods. In addition, the method treats the data in such a way as to give more weight to the higher time data than to the lower time data.

Several examples of the use of the method are described in detail in the body of the paper, and the results of this type of extrapolation on the creep-rupture curves of several materials are presented. The use of this method is, of course, not restricted to creep-rupture data, and several examples of other type curves such as Bessel functions, Legendre polynomials, and power functions, are presented.

From authors' summary by M. A. Jaswon, England

4626. Taira, S., and Koterazawa, R., Prediction of dynamic creep strength under combined stress of static tension and repeated bending, using static creep data, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15-16, 1959; Japan Soc. Test. Mat., 1960, 68-71.

Expressing the strain rate in the strain-hardening form, authors give an analytical solution for predicting dynamical creep of material under axial pulsating stress, if static creep data are known. Title problem is then solved, assuming that the creep strain, as well as the total strain, may be regarded as constant over the entire cross section and that the plastic component of alternating strain is negligible.

Comparison of the results obtained by analysis with those obtained by experiment was carried out for cases of a low-carbon steel and a 13 chromium steel. The results of the earlier investigation have been previously presented [AMR **13**(1960), Rev. 6342].

C. Hoschl, Czechoslovakia

4627. Kuratov, P. S., and Rozenblium, V. I., On the integration of the equations of unsteady creep of solid bodies, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24,** 1, 195-199, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Authors show how the analysis of creep of a body, where boundary stresses are given on part of the surface, and velocity vector on part of it, can be reduced to an isothermal problem of the theory of elasticity with additional surface- and body-loadings. The magnitudes of these depend on the creep law under consideration. This reduced problem can be solved numerically—in which case the creep law may be chosen rather freely—or in special cases analytically.

A. Isaksson, Sweden

4628. Rozovskii, M. I., Certain properties of special operators used in the theory of creep, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 5, 1398-1402, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Concerning Volterra's hereditary theory, the author deals with the relaxation operator deduced by Rabotnov [*Prikl. Mat. Mekh.* **12,** 1, 1948] from the creep operator with Abelian nucleus. Two properties which define the same operations with these operators are given. The results are used to obtain the stress state of a shaft made up of m cylindrical nonhomogenous and anisotropic layers.

M. J. Predeleanu, Roumania

4629. Rozenberg, V. M., Evaluation of the inclusion of the grain boundaries in the general deformation due to creep (in Russian), *Zavod. Lab.* **25,** 1, 53-57, 1959; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12748.

The determination of the deformation due to creep along the boundaries of the metal's grains is ordinarily performed by etching a network of graduation lines. The transposition of the lines is measured usually on 100-200 (and sometimes on 500) boundaries and is averaged. The precision of this method depends to a large degree on the orientation of the boundaries relative to the sample's axis. Data are given to show the distribution of the grain boundaries at the angles with the axis of the sample. The defects in the method of etching a network of lines are pointed out and a description is given of a more accurate method which takes into consideration all three components in the region of the vector transposition of the grain. This transposition vector is determined by the optical method of the disruption of the bands of interference on the grain boundaries with the aid of formulas obtained by D. McLean. A method is also briefly touched on for the determination of the deformation due to creep on the boundaries of the grains by observing the changes in the form of the grains.

I. L. Kats

Courtesy Referativnyi Zhurnal, USSR

4630. Biot, M. A., Instability of a continuously inhomogeneous viscoelastic half-space under initial stress, *J. Franklin Inst.* **270,** 3, 190-201, Sept. 1960.

Author uses a rather general linear viscoelasticity theory of initially stressed, inhomogeneous semi-infinite body to analyze surface folding associated with dynamic instabilities.

J. L. Ericksen, USA

4631. Kasimov, A. F., Displacement of one viscoplastic liquid by another (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Neft'i Gaz.* no. 6, 97-103, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11914.

The problem is investigated of the displacement of a heavy viscoplastic liquid by another in the case where these liquids, of different specific gravities, are in motion in a vertical round tube when the pressure along the tube is falling longitudinally. The viscosity coefficients and the limits of shear stress in both the liquids are assumed to be identical. The author in his solution disregards the inlet portion of the tube and thinks it possible to assume that the distribution of velocities on the line of contact would be the same for each of the liquids as would be the case were only one liquid in motion in the tube. Having made these assumptions the author proceeds to determine in a simple calculation the time of emergence of the liquid being displaced in the end section of the tube and the relation of the volume of the displacing liquid remaining in the tube to its initial volume.

S. M. Targ

Courtesy Referativnyi Zhurnal USSR

Plasticity

(See also Revs. 4613, 4649, 4674, 4723, 4777, 4798, 4808)

4632. Cowper, G. R., The elastoplastic thick-walled sphere subjected to a radial temperature gradient, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 496-500, Sept. 1960.

Author obtains exact solution for ideal elastoplastic sphere thermally stressed by slowly increasing temperature difference between inner and outer surfaces, assuming yield condition independent of temperature. Yield occurs first at inner boundary. A second yield zone later spreads inward from outer boundary if the ratio of boundary radii is less than 2.791 (the only case for which complete solution is given), but forms in the interior for greater ratios.

An approximate analysis neglecting elastic strains is also presented; this solution involves a surface of discontinuity of circumferential stress where there is a jump from compression inside to tension outside the discontinuity surface. The approximate analysis gives the asymptotic form of the exact solution at infinitely high temperatures. Author states that the approximate analysis is readily extended to stress calculation where yield de-

pends on temperature, but the displacement calculation is more difficult. L. E. Malvern, USA

4633. Hodge, P. G., Jr., and Sankaranarayanan, R., Plastic interaction curves for annular plates in tension and bending, *J. Mech. Phys. Solids* **8,** 3, 153–163, Aug. 1960.

An annular plate made of perfectly plastic material is subjected to a uniform normal load over its surface and a constant tensile load around its outer edge. Close bounds on the interaction curve are found for all values of the inner radius under various support conditions at the inner edge.

From authors' summary by A. Boresi, USA

4634. Shield, R. T., On the optimum design of shells, *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 316–322, June 1960.

A procedure is developed for obtaining the design of a shell (or other structure) which is optimum for a given criterion, e.g. for the minimum-volume requirement. It is assumed that the material of the shell is elastic-perfectly plastic. The loading forces are given. The action of body forces is included in the analysis. The procedure is obtained by using the lower-bound theorem and upper-bound theorem of D. C. Drucker, W. Prager and H. J. Greenberg [AMR **5,**(1952), Rev. 1965] and not by direct application of the calculus of variations. For illustration, some problems are solved for the particular case of the minimum-volume design of a circular cylindrical sandwich shell. The results indicate that only for relatively short shells does the minimum-volume design effect an appreciable saving over either the design with constant thickness or the membrane design.

J. Barta, Hungary

4635. van Rooyen, G. T., and Backofen, W. A., A study of interface friction in plastic compression, *Inter. J. Mech. Sci.* **1,** 1, 1–27, Jan. 1960.

Experimental study of the distribution of friction includes measurement of normal pressure, shear stress, and friction coefficient. Experimental technique uses pressure-sensitive pins installed in the platen. Tests carried out to determine the friction hill were conducted on aluminum, copper and iron. Lubrication included lead foil, molybdenum disulphide, soap, teflon, mineral oil, and an absence of lubrication.

An extensive series of tests with increasing amounts of plastic deformation are reported. The results show that the analysis by Seibel is applicable only for conditions of low coefficient and small reduction in thickness. Deformation becomes nonhomogeneous and lubricant film breaks down at the periphery at increased reduction in thickness.

The work demonstrates that the technique of measurement can provide rather clear insight into lubrication and friction in plastic-working operations. J. E. Stallmeyer, USA

4636. Malyshev, B. M., Plastic flow during joint continuous tension and torsion under the action of small torsional moments (in Russian), *Vestn. Mosk. In-ta Ser. Mat., Mekh., Astron., Fiz., Khim.* no. 1, 55–68, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12305.

The applicability is determined experimentally of the theory of plastic flow in the case of continuous tension, to which very small stresses due to shear are added. An explanation is furnished for the direction of the flow. The experiments were carried out on different materials in a wide range of deformations, shear stresses and velocities of deformation. An ordinary machine for testing for rupture was used for the experiments, the machine being adapted for dealing with tests with complex loadings. Tests were carried out on brass tubes and bars, steel bars, aluminum bars and other materials. An analysis of the experimental data enables the following deductions to be drawn: (1) A material introduced into the plastic region, with the application of a force as small as may be desired acting in a direction differing from the original direction, is endowed with a plastic flow the velocity of which is proportional to the force applied and to the velocity of the deformation in the first direction. (2) Any divergence from the coaxial condition depends only on the deformation and does not depend on the velocity of the deformation of the tension material and on the shear stresses. This deduction was obtained from the experiments on the brass test samples. (3) For copper a significant divergence from the coaxial condition was observed at loads which were close to being simple loads.

V. A. Sveshnikova

Courtesy Referativnyi Zhurnal, USSR

4637. Tseitlin, V. Z., Evaluation of the plasticity of heat-resistant alloys when being fractured (in Russian), *Zavod. Lab.* **25,** 1, 87–90, 1959; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12752.

The evaluation of plasticity in materials of low plasticity, which occurs when the waist of the sample is fractured, by observing the relative elongation may lead to large errors. The results of tests to determine the long-term strength of a steel mark EI 765 at 565–650° show that the change in the magnitude of the relative elongation at varying times for the soaking takes place within wide limits. At the same time the change of magnitudes of relative constriction takes place within much more restricted limits. Taking the above into consideration a recommendation is put forward to avail oneself of the more regular characteristic of plasticity—the relative constriction, another factor being that the relation of the length to the diameter of the test sample and the formation of transverse cracks in the sample during the tests may exercise a marked influence on the magnitude of the relative elongation when fracture takes place. This characteristic is convenient for use in working over tests for long-term strength and for creep expressed in coordinates of actual stress-relative constriction. I. L. Kats

Courtesy Referativnyi Zhurnal, USSR

4638. Taira, S., and Murakami, Y., On the residual stresses due to alternating stresses of annealed steels, *Bull. JSME* **4,** 13, 41–45, Feb. 1961.

During alternating stressing of annealed steels in reversed bending, residual compressive stress is introduced near the surface of specimen, and its magnitude changes with the number of stress cycles. Residual stress appears from the very early period of stress repetitions, and the maximum value occurs at about 10^5 stress cycles, thereafter it fades gradually. Residual stress was found to vanish approximately immediately before fracture. The higher the carbon content or the yield point of the steel, the larger is the residual stress produced. The change in hardness due to alternating stressing is quite similar to that in residual stress. In the case of ascending stressing, both residual stress and hardness were found to increase more than in the case of stressing at a primary stress amplitude. By numerical calculations, the contribution of residual stresses to the fatigue strength at a fixed stress amplitude was studied. On the basis of this ideal *S-N* curve, the role of residual stresses was also made clear.

From authors' summary

4639. Dehlinger, U., Formation of residual stresses in metal (in German), *Z. Metallk.* **50,** 3, 126–130, Mar. 1959.

4640. Fitzgerald, E. R., Mechanical resonance dispersion and plastic flow in crystalline solids, *J. Acoust. Soc. Amer.* **32,** 10, 1270–1289, Oct. 1960.

An explanation is given for the mechanical resonance dispersions observed at audio-frequencies in many crystalline solids.

The resonances are considered to result from alternating simultaneous slip occurring on transient slip planes with mean lifetimes of 10^{-3} to 10^{-4} sec. The slip along such planes is opposed by displacement-dependent, velocity-dependent, and acceleration-dependent stresses. These last two stresses are found to be of magnitudes which can result from the interchange of atoms across a slip plane with a consequent decrease of momentum of the fast moving (slipped) part of the sample undergoing deformation. If the rate of transfer is independent of the slip process, a velocity-dependent (viscous) stress results, but if some of the atoms are induced to transfer as a direct result of the slip itself, then an acceleration-dependent stress is present. These slip planes, because of their sudden formation, short lifetimes, and inertial coefficients, do not contribute to the static or ultrasonic compliance, but do affect the dynamic compliance at audio-frequencies. The transient slip planes are supposed to be generated by the movement of dislocations across the sample. Thus, a concept of combined consecutive and simultaneous slip is suggested in which dislocations remain an essential part of the mechanism but are not the sole source of deformation. An interesting consequence of the theory is that a critical stress for plastic flow or, alternatively, for brittle fracture can be predicted from values of the dynamic compliance (or modulus). Predicted values of critical stress (for both single crystals and polycrystalline metals) made from the dynamic modulus at 100 cps are much closer to the observed values than those made on the basis of either the static or ultrasonic moduli. In particular, it is found that the predicted values for brittle fracture stress agree almost exactly with observed breaking stresses for single crystals of sodium chloride, germanium, glycine sulfate, and, at two temperatures and orientations, for natural quartz crystals. The occurrence of negative absorption or induced emission is discussed, and a means of generating sustained, self-excited, audio-frequency oscillations in a crystalline solid is outlined. The possibility of mechanical failure at an intrinsic material resonance is considered briefly and some other aspects of the mechanical behavior of crystalline solids are also examined. From author's summary

4641. Rabotnov, Iu. N., Model illustrating some properties of a hardening plastic body, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 1, 219–228, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

Rods, Beams and Strings

(See also Revs. 4618, 4621, 4622, 4678, 4697, 4698, 4700, 4705, 4714, 4777)

4642. Oplinger, D. W., Frequency response of a nonlinear stretched string, *J. Acoust. Soc. Amer.* **32,** 12, 1529–1538, Dec. 1960.

The first-term approximation (elliptic cosine in time and trigonometric functions in space) to the title problem is given. Numerical examples of frequency response curves for elliptic cosine driving at one end of the string are plotted for several driving amplitudes.

Experimental results are presented for a string driven sinusoidally at one end by an electrodynamic shaker. Detuning of out-of-plane motions using a slot to confine motion near the fixed end of the string (giving it a larger apparent length for out-of-plane motion) was successful enough to allow comparison of theory with experiment. Observations of both frequencies and amplitudes of the hard-spring jump phenomenon compared well with the theory. Observed string tension did not agree as well with theory, particularly as the frequency was increased well up the backbone curve. The sharper rise of observed tension was attributed to considerable out-of-plane motions at these large amplitudes. Also, during these large tension oscillations, the base (minimum) tension was observed to decrease. The author suggests that this is due to viscoelastic effects, but presents no figures to show that reasonable assumptions for his nylon string are of the right order of magnitude to explain the decrease.

H. E. Lindberg, USA

4643. Zapalowicz, W., The torsion of regular polygonal bars—Computation formulas (in Polish), *Rozprawy Inz.* **8,** 4, 671–677, 1960.

Computation formulas are given for the torsional rigidity and the maximum shear stress during free torsion of prismatic bars with the cross section in the form of a regular n-sided polygon. The final form of the equations is

$$D = \alpha G I_0, \qquad \tau_{max} = \alpha_1 G \beta a = \frac{M_s}{\alpha_2 a^3}$$

where I_0 is the polar moment of inertia of the cross section, a the radius of the circle circumscribed on the polygon and α, α_1 and α_2 numerical coefficients depending on the number of sides n. To find these the author uses the results obtained by himself in the paper "Torsion of prismatic bars of regular polygonal cross section" [*Arch. Mech. Stos.* **11,** 5, 559–590, 1959; AMR **13**(1960), Rev. 3296] where expressions for torsional rigidity and maximum stress are obtained in the form of infinite series.

In the present paper numerical tables are given for the coefficients α, α_1 and α_2 for various n. The upper bounds of the errors of these coefficients are determined in an approximate manner as sums of a finite number of terms of the relevant infinite series.

The results are presented in a form convenient for application. They may be used by both scientific workers and engineers.

J. Leyko, Poland

4644. Shateav, V. G., Evaluation of the local strengthening of the longitudinal ribs in cylindrical thin-walled beams (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Aviats. Tekhn.* no. 2, 64–73; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12241.

An approximate solution is given of the stressed state in a cylindrical round shell reinforced by longitudinal rigid ribs and when under the action of longitudinal forces, distributed in arbitrary fashion on the perimeter of the shell. Some of the ribs have a varying transverse sectional area which is taken into account by the author. When solving the problem the stressed state in the shell is divided into (1) a so-called initial portion and (2) a "supplementary" portion which is distinguished by the variability of section in a number of the ribs according to their length. The numerical calculations, for which the experimental data are used, are given; here, the calculated and experimental values for the stresses show good agreement. V. A. Postnov

Courtesy Referativnyi Zhurnal, USSR

4645. Veselovskii, G. V., Calculation for a continuous beam with a broken axis (in Russian), *Trudi Novocherk. Politekhn. Inta* **42/56,** 26–53, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12402.

An exposition is given of the theory on the calculations for continuous beams with a plane multi-angled axis, inscribed in the arc of the periphery, when under the action of loads perpendicular to the plane in which the axis of the beam is situated. For the basic unknown two moments are taken in sections perpendicular to the axis of the beam and passing through the bisectreces of the angles between the axis of two adjacent spans in contact with the whole of the intermediate even number supports. For the basic system obtained, consisting of two-span beams (*see* F. D. Dmitriev,

Inzbener. Sbornik **5**, no. 2, 1949), six-term equations are furnished for the transpositions by the method of forces, for cases of even and odd number of spans of the given continuous beam. Formulas are derived for the determination of the individual angles of turn and the torsion of the bisectoral support sections and also for the loading terms when certain forms of loading are operating. An investigation is made of the case where the ends of the beam are clamped and also of the calculations for plane multiangled rings (clamped continuous beams). A. A. Gorin

Courtesy Referativnyi Zhurnal, USSR

4646. Shkenev, Yu. S., Nonlinear vibrations of circular rings (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **28**, 82–86, 1960.

Author assumes the deflections are small compared with the radius of curvature, but are of the order of the thickness of the ring. The ring is inextensible. Author considers an approximate solution with the help of the principle of virtual work. The solution for free vibrations with initial displacements given by a cosine function is expressed in terms of elliptical integrals. Forced vibrations are considered in the case of uniformly distributed radial load of cosine time dependence. The amplitude of the deflection is calculated under the assumption that the period of the deflection corresponds to the period of the load, thus higher components of the motion are omitted.

There are no numerical examples and in the case of free vibrations there is no comparison with the linear solution.

P. Wilde, Poland

Plates, Shells and Membranes

(See also Revs. 4614, 4633, 4634, 4683, 4688, 4690, 4695, 4703, 4704, 4706, 4707, 4708, 4719, 4724, 4780, 4803, 4938, 5069, 5156)

4647. Herber, K.-H., Elastically supported plate with strip loading, particularly bottom plating under interior supports of tanks (in German), *Bautechnik* **38**, 2, 47–50, Feb. 1961.

Paper gives the solution for plates on elastic foundation loaded by long rigid bodies of constant width "a." Simple formulas and curves for calculating maximum stresses are given, which are valid when "a" is smaller than a certain limit. For larger values of "a," an approximate procedure is described which can be easily used with the aid of a numerical table. A case of strip loading on a cylindrical shell is briefly discussed.

The results obtained in the paper may be used for calculating the stresses in bottom plates of steel tanks loaded by internal supports. C. E. Massonnet, Belgium

4648. Nasitta, Kh., The thin, simply supported annular plate subject to concentrated load (in German), *ZAMM* **41**, 1/2, 31–43, Jan./Feb. 1961.

Author treats the problem of an annular plate, loaded by a transverse point force, by classical thin-plate theory. Outer circular boundary is simply supported and inner boundary is free. Method of solution consists of generalization of the known Green's function for the deflection of a simply supported circular plate so as to permit satisfaction of boundary conditions on the free inner boundary. Results are very involved and numerical deflection evaluation for specific loading necessitates use of electronic computer. No mention is made of bending moments and shear forces. Reviewer does not believe that author's method is suitable for evaluation of these. Z. Hashin, USA

4649. Mazing, R. I., Influence of the form of an impulse on deflections of a circular plate (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 6, 143–146, Nov./Dec. 1960.

This short paper studies the effect of the impulse shape on the deformation of an elasto-plastic circular plate. By considering permanent deformations of the plate resulting from rectangular and sinusoidal impulses of varying amplitude and duration, it is concluded that the final deformation of the plate is independent of the pulse shape. The solution is carried out with the use of Tresca's plastic yield criterion. V. Chobotov, USA

4650. Solecki, R., General solution for a plate having the form of a right-angled triangle (in Polish), *Rozprawy Inz.* **8**, 2, 203–210, 1960.

This is a generalization of the author's former paper [General solution for the 30°–60°–90° plate by means of eigentransforms, *Arch. Inżyn. Lądow.* **6**, no. 2, 1960]. The object of the considerations is a plate in the form of a right-angled triangle resting on a Winklerian foundation and loaded in its plane with uniform compression forces and satisfying on each edge any continuous boundary condition. The proper (eigen) transformation is defined thus

$$T_\phi\,[f(x,y)] = f_\phi - (m,n) = \iint_F f(x,y)\quad \phi_{mn}(x,y) - dF,$$

where F is the region of the plate under consideration and $\phi_{mn}(x,y)$ is an eigenfunction satisfying the differential equation

$$\nabla^2\,\nabla^2\,\phi_{mn} - k^4_{mn}\qquad \phi_{mn} = 0$$

and the boundary conditions $\phi_{mn} = \nabla^2\,\phi_{mn} = 0$. The inverse transformation is defined thus

$$T_\phi^{-1}\,[f_\phi\,(m,n)] = f(x,y) = \sum_{m,n} \frac{f\phi\,(m,n)\,\phi_{mn}(x,y)}{\iint_F [\phi_{mn}(x,y)]^2\,dF}.$$

With these definitions the general solution is given for any right-angled triangle. It will be of purely formal character unless the eigenfunctions are known. The author explains the physical sense of the mathematical operations performed.

The plate having the form of an isosceles rectangular triangle is considered in more detail, the eigenfunctions being given. The author's method enables the solution of a number of new problems and also the verification of the existing solutions obtained by the reviewer in another way [Free vibrations and buckling of a triangular plate, *Arch. Mech. Stos.* **8**, 14–28, 1956; AMR **10** (1957), Rev. 1012]. Ten examples concerning triangular plates with various boundary conditions are solved by obtaining infinite systems of linear algebraic equations. Setting the principal determinant of each of these systems equal to zero enables us to find the natural frequency of the plate under consideration or the critical load. Reviewer notes it can easily be shown that if, at two edges of the plate $x = 0$ and $r \pm 0$ intersecting at the angle φ, the boundary conditions $\Phi = \nabla^2\Phi = 0$ are satisfied we must have in the vertex

$$\frac{\partial^2\Phi}{\partial x \partial y}\cos^2\varphi = 0.$$

It follows that at sharp angle vertices, neither torques nor concentrated reactions can appear in the case of bending according to an eigenfunction. This observation enables a simplification of the Eqs. [2, 6], [3, 2], [3, 4] and [3, 5] and Fig. 2c.

Z. Kaczkowski, Poland

4651. Czudek, H., Some problems of strength of an infinite plate with a rigid inclusion (in Polish), *Rozprawy Inz.* **8**, 3, 603–626, 1960.

The problem is that of an infinite plate with rigid inclusion of which the outside can be mapped conformally onto the outside of a unit circle by means of the known function.

$$z = \tilde{\omega}(\xi) = K\left(\xi + \frac{m}{\xi^n}\right)$$

The stresses are found for loads of two types. The first load type is that of uniform tension p; the second results from cooling the rigid inclusion (previously heated), the temperature difference being t.

The starting point is the displacement of the contour of the inclusion in agreement with the type of load. Starting from the equations for displacements (used by F. Szelagowski for the solution of a similar problem for an elliptic inclusion), the stress functions $\phi(\xi)$ and $F(\xi)$ are obtained.

The stress thus obtained can, in the author's opinion, be used for the determination of the order of magnitude of the effective stress in sheet-metal elements subjected to tension and in the case of thermal stresses produced by cooling plates welded on a sheet metal element, and also to find the most convenient form of such a plate from the point of view of stress concentration.

Assuming $n = 1, 2, 3$ and considering three values $0 \leqslant \frac{1}{n}$,

equations are given, for both load types, for the stresses in the plane of the unit circle for a rigid inclusion in the form of a circle, an ellipse, a line segment and curvilinear triangles and squares of three types.

In the latter case the tension in the direction parallel to the diagonal and at the angle 45° is considered.

To obtain the degree of stress concentration at the characteristic points of the contour the stresses are calculated for a steel plate by substituting the coordinates of point of the unit circle $\xi = i$ and $\xi = 1$, although the stress functions have at this point for $m = \frac{1}{n}$, for instance, $(n+1)$-fold poles and the contour curves of the inclusions constitute branch curves of the mapping function.

W. Zapalowicz, Poland

4652. Solecki, R., The general solution of a triangular plate 30°—60°—90° by means of eigentransform (in English), *Bull. Acad. Polonaise Sci. (IV)* **8**, 7, 325–332, 1960.

Paper is an extension of Wilde's application of finite sine Fourier transforms in general investigations of rectangular plates [AMR **12** (1959), Rev. 4930]. Also see AMR **14** (1961), Rev. 1858. The kernel of the transform is the eigenfunction of the vibrating simply-supported plate.

The problem under consideration is described by a nonhomogeneous differential equation for amplitudes of vibrations of a uniformly compressed isotropic plate on elastic foundation. In greater detail are considered a simply supported plate and a plate clamped along the longer cathetus and simply supported along the remaining two sides.

A more extensive study of the problem is contained in author's paper published under the same heading in *Archiwum Inżynierii Lądowej* **6**, 2, 1960.

R. Schmidt, USA

4653. Kacner, A., Method of two fundamental systems in bending of plates with discontinuous boundary conditions (in English), *Bull. Acad. Polonaise Sci. (IV)* **8**, 7, 351–360, 1960.

Author considers arbitrarily loaded semi-infinite plate strips, simply supported at the long edges, with various conditions of discontinuous support at the short edge. Green's functions are used to pass from the fundamental systems (both of them with homogeneous boundary conditions along the entire short edge) to the system with discontinuous boundary conditions. The analytical results obtained are in complete agreement with those of AMR **12** (1959), Rev. 1763.

R. Schmidt, USA

4654. Vlasov, V. V., Application of the method of initial functions to certain problems in the bending of rectangular plates (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **30**, 78–84, 1960.

The solution follows the classical procedure of superimposing a particular solution of the nonhomogenous problem upon the solution of the homogenous problem. The particular solution is obtained using trigonometric sine series.

J. Solvey, Australia

4655. Slobodyanskii, M. G., Bending of a plate of variable thickness (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mashinostroenie* no. 5, 99–108, 1959.

Problem of bending of an isotropic plate of variable thickness is defined: first as a boundary-value problem for partial differential equation of fourth order with variable coefficients, and boundary conditions of built-in and simply supported edges; secondly as a variational problem for a quadric functional expressing the value of potential energy of deformations of the plate, and the same boundary conditions.

The cases of continuously distributed load with intensity p, (the solution denoted by $w(x, y)$), and of concentrated load $p_1 = 1$ at the point Q (the solution denoted by $w_1(x, y)$) are simultaneously investigated. The solutions of both w and w_1 are sought in the form of a linear combination of coordinate functions φ_i satisfying the given boundary conditions:

$$w(x, y) \doteq w_n = \sum_{i=1}^{n} \alpha_i \varphi_i,$$

$$w_1(x, y) \doteq w_{1n} = \sum_{i=1}^{n} \beta_i \varphi_i$$

The values of parameters α_i and β_i respectively are then determined from the minimum condition of two functionals derived by certain transformations from the given functional.

Further, the expressions for bending moments of the plate are sought. This question is solved by means of a Green's function $G(x, y, \xi, \eta)$ of the given boundary-value problem, which is taken in the form

$$G(x, y, \xi, \eta) = g_0(x, y, \xi, \eta) + g(x, y, \xi, \eta).$$

Here $g_0(x, y, \xi, \eta)$ denotes a special main part of Green's function G. In conclusion a method for determining the function g_0 for the considered plate is given.

V. Kovarik, Czechoslovakia

4656. Ventskovskii, B. K., The theory of deflection of ring-shaped plates under the simultaneous action of transverse and radial forces (in Russian), Raschetyi na Prochnost', Vol. 2, Moskva, Mashgiz, 1958, 94–123; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12258.

The axisymmetrical deflection of ring-shaped plates is investigated when the plates possess an arbitrary number of force segments, different transverse loads and are under the action of radial loadings on the contour. The solution is arrived at in Bessel functions and has the form inherent in the method of initial parameters. Tables were compiled to lighten the burden of calculation for the function of influence. Two numerical examples are examined.

S. N. Sokolov

Courtesy Referativnyi Zhurnal, USSR

4657. Ventskovskii, B. K., The deflection of ring-shaped and round plates on a generalized elastic base under the simultaneous action of transverse and radial forces (in Russian), Raschetyi na Prochnost', Vol. 3, Moskva, Mashgiz, 133–150, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12259.

A round symmetrically loaded plate on an elastic base conforming to the Winkler hypothesis is the subject of investigation. The base also gives a moment reaction proportional to the angle of turn of the normal to the surface of the plate. A differential equation for the deflection of the plate is derived and examples are given of its integration. V. L. Biderman

Courtesy Referativnyi Zhurnal, USSR

4658. Zhilyakov, Ya. G., A practical method for calculations for plates carrying a static load (in Russian), *Trudi Khar'kovsk. Inzh.-Stroit. In-ta* no. 7, 11–20, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12261.

The solution of the deflection of a rectangular plate with dimensions of $a \times b$ with free and supported faces is sought in the form of

$$w(x, y) = \psi(y)\, f(x) + F(y)\ \varphi(x)$$

$$f(x) = \sum_{n=1}^{\infty} \cos \frac{n\pi x}{a}, \qquad F(y) = \sum_{n=1}^{\infty} \cos \frac{n\pi y}{b}$$

The introduction of these expressions in the differential equation for the distorted mean surface enables an expression to be found by means of the assigned principle for the distribution of the transverse load for the functions $\psi(y)$ and $\varphi(x)$. The constants of the integration are determined from the boundary conditions for the support of the plate. For edges which are freely supported the boundary conditions are satisfied with precision, while for free and clamped edges, in the integral form in the sense of the Bubnov-Galerkin method. The comparison of the results of the calculations made with the expressions for $w\,(x, y)$ obtained by the author with B. G. Galerkin's results ["Collection of works," Moscow, Izd-vo Akad. Nauk SSSR, 1953] shows that the proposed method proves to be sufficiently accurate for the determination of forces. V. P. Belkin

Courtesy Referativnyi Zhurnal, USSR

4659. Burmistrov, E. F., On the problem of stress concentration near noncircular holes in thin plates subjected to bending (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **30**, 99–106, 1960.

Author investigates an infinite plate with three types of holes: oval, rectangular with semicircles at two ends and triangular (isosceles triangle with an angle of 30°). The loading at infinity is either bending in one direction, bending in two directions or torsion. The solution is based on Muskhelishvili's method, and the numerical nondimensional results for each type of hole and loading are given in tables. J. Solvey, Australia

4660. Bilevich, A. F., Calculations for round plates loaded axisymmetrically, Part 1: Plates which are statically determinable in relation to the supporting reaction (in Russian), *Sb. Naucbn. Tr. Tomskii Inzb.-Stroit. In-ta* **3**, 179–200, 1957; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12262.

Formulas are derived for the calculation of the deflection moments and transpositions (angular and linear) in a statically determinable continuous round plate of constant thickness, which is under the action of several transverse forces distributed evenly over the concentric surrounding region. In the case of a symmetrical surface, transverse load and moment distributed evenly over the surrounding region, the relevant formulas are derived by the method of limited transfer. Tables are given for the calculation coefficients when the value of Poisson's coefficient is equal to 1/6 (ferro-concrete) and examples of the computations are furnished. An investigation is made of the calculation for a statically indeterminable round plate with a clamped contour or with several concentric supports under the action of the above-mentioned load; here, in order to determine the reactions in places possessing surplus linkages, use is made of the canonical equations for the method of forces.

Abstractor's remarks: The reading of this paper is made difficult by numerous errors in the formulas and inexact expressions. For instance, the deflection moment per unit of length of the section across the diameter is called the tangential deflection moment coming into the ring-shaped section per unit of length; transpositions are called deformations. It should also be mentioned that the author has not taken into consideration the known generalized solutions for round plates worked out earlier by S. N. Sokolov [*Tr. Mosk. In-ta Khim. Masbinostroeniya*, no. 1, 1935]. A. D. Kovalenko

Courtesy Referativnyi Zhurnal, USSR

4661. Bondel, H., Static behavior of spherical concrete shells with reinforced or prestressed concrete edge ring in the case of rotationally symmetric loading taking into account creep and shrinkage (in German), *Bauingenieur* **36**, 2, 46–54, Feb. 1961.

It is well known that discontinuity forces and moments occur under load in a concrete spherical shell between the shell itself and its stiffening ring, due to compatibility conditions. The paper studies the influence which different dimensions and loose reinforcement or prestressing of this ring exert on the magnitude and shape of these forces. In particular, the consequences of shrinkage and creep deformations are studied in detail, by extending to the present problem an approximate method developed by the author. The calculations take into account a variation in thickness of the edge zone of the shell.

The paper concludes with a very interesting numerical comparison between ten different constructional possibilities; the corresponding diagrams of internal forces and moments are given and discussed in detail. It is shown that very large errors on the stresses are usually made when a prestressed ring is used, if one ignores the effect of shrinkage and creep. Reviewer considers the paper as an important step toward a thorough understanding of the actual behavior of concrete structures. C. E. Massonnet, Belgium

4662. Apeland, K., Stress analysis of translational shells, *Proc. Amer. Soc. Civ. Engrs.* **87**, EM 1 (*J. Engng. Mech. Div.*), 111–139, Feb. 1961.

This paper is concerned with the problem of structural analysis of thin shallow shells in the form of a translational surface, the middle surface of which is defined in Cartesian coordinates by

$$z(x, y) = (1/2)\, k_2\, (\gamma x^2 - y^2) \qquad [1]$$

in which $\gamma = k_1/k_2$ is a shape parameter. Depending of the sign of γ, Eq. [1] represents a hyperbolic paraboloid ($\gamma > 0$), a parabolic cylinder ($\gamma = 0$) or an elliptic paraboloid ($\gamma < 0$).

Marguerre equations in terms of normal displacement w and stress function F are reduced to a single eighth-order equation for a new stress-displacement function ϕ defined by

$$w = \nabla^4 \phi; \quad F = E\, b\, k_2 (\phi_{,xx} - \gamma \phi_{,yy}).$$

Levy-type solutions of this equation are obtained. The solutions are expressed in terms of two parameters, the shape parameter γ and a shell parameter ε. The solutions are developed explicitly and a systematic procedure for the application in solving actual boundary-value problems is developed. A number of tables of the roots of the characteristic equation are included. It is shown that tables analogous to those of Rüdiger-Urban for cylindrical shells may be calculated with the aid of an electronic computer.

Some practical applications are made, showing that the damping of edge disturbances is slower in hyppar shells than in cylindrical or elliptic paraboloid shells. C. E. Massonnet, Belgium

4663. Bieger, K. W., The circular cylindrical shell subject to concentrated loading (in German), *Ing.-Arch.* **30**, 1, 57–62, Jan. 1961.

In order to determine the state of stress in an isotropic, homogeneous, thin circular cylindrical shell subjected to concentrated forces, author expresses displacement components u, v, w in terms of a Pucher type of stress function, F. The stress function is introduced for three separate, independent cases; viz, when internal loads, X (longitudinal), Y (circumferential), Z (radial) are (1) $Y = Z = 0$; (2) $Z = Y = 0$; (3) $X = Y = 0$. Components u, v, w are then substituted into Flügge's three coupled differential equations relating displacements to internal loads. This gives an eighth-order differential equation of equilibrium with three different inhomogeneous parts; X, Y, Z, corresponding with the three cases above. Using Maxwell-Betti's law of reciprocal deflections, author shows v for $(X = 1) = u$ for $(Y = 1)$; w for $(X = 1) = u$ for $(Z = 1)$; and w for $(Y = 1) = v$ for $(Z = 1)$.

Author works the case of a radially concentrated group of loads by considering an infinitely long shell subjected to two self-equilibrating radial forces. He represents Z as the sum of a Fourier series and a Fourier integral. F is represented similarly with coefficients which are determined from the numerical solution of an eighth-order algebraic equation. Influence surfaces are drawn for bending moment for two values of the parameter k (one-twelfth of the square of the ratio of shell thickness to radius). Author claims boundary conditions may be quickly matched using tables compiled by Rüdiger and Urban. From the influence surface the state of stress for any other appropriate loading condition may be found by Maxwell's law. R. W. Shreeves, USA

4664. Houghton, D. S., and Johns, D. J., Deformation equations for non-circular cylinders, *J. Roy. Aero. Soc.* **64**, 600, 765–766 (Tech. Notes), Dec. 1960.

Explicit deformation equations for noncircular cylinders subject to lateral pressure are derived following the general derivations in "The Theory of Thin Shells" by Novozhilov [P. Noordhoff Ltd., 1959; AMR **13** (1960), Rev. 2749]. The equations are reduced to the special case of a circular cylinder for the purpose of comparing with results in standard references. It is found that they differ slightly. No effort is made to ascertain the implications of the differences. The characteristic equation for circular cylinders subject to boundary loadings is also given but no comparison is made with available references. D. H. Cheng, USA

4665. Csonka, P., On the stress function of the circular cylindrical shell (in English), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **29**, 1/2, 87–98, 1960.

Author applies, to several special cases and approximations, his previous solution (in same journal in 1953) of Flügge's cylinder equations with zero Poisson's ratio, using a "displacement function" related to the displacements u, v, w. No reason is given for use of this approach rather than that of "uncoupling" the equations by solving for u and v in terms of w and obtaining an equation in w alone, which seems to accomplish the same purpose much more simply. L. H. Donnell, USA

4666. Csonka, P., Paraboloid shell of revolution over an equilateral triangle base (in English), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **29**, 3/4, 313–332, 1960.

Membrane stress distribution is calculated for a shell as specified in the title. A series of stress functions, which satisfy the boundary conditions and correspond to axially symmetric loading, are used to approximate to practical loading conditions. A combination of at most two such functions is shown to be adequate in a numerical example. W. S. Hemp, USA

4667. Galimov, K. Z., The formulation of geometrical boundary conditions for the nonlinear theory of shells in forces and moments (in Russian), *Izv. Kazansk. Fil. Akad. Nauk SSSR, Ser. Fiz.-Mat. i Tekhn. Nauk* no. 12, 17–27, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12226.

Author develops his earlier studies [*Izv. Kazansk. fil. Akad. Nauk SSSR, Fiz. Mat. i Tekhn. Nauk* no. 10, 3–26, 1956; *Uch. Zap. Kazansk. In-ta* **116**, 1, 36–40, 1956] by making use of the procedures of tensor analysis and of the theory of surfaces and obtains a number of correlations for the nonlinear theory of shells. A detailed examination is carried out of the case of small increases in length and of shear at arbitrary angles of twist. Author establishes direct equations for the cooperative action of forces and moments and for the natural boundary conditions formulated in terms of components of the deformation of the boundary element. In the first of the two examples investigated it was shown that the geometrical boundary condition of the fastened edge corresponds to the deformation condition of the rigid edge. It is not indicated that these conditions do not appear to be complete equivalents. Conviction regarding this fact is easily effected by an investigation, for instance of the problem on the deformation of a closed cylindrical shel. reinforced along the edges by rigid flanges adjusted in regard to each other. What has been stated refers as well to other problems where there are predominantly geometrical boundary conditions. Making use of deformation boundary conditions introduces appreciable simplifications when shells are subjected to elastic interlinkage, as this obviates the necessity to determine the transpositions first. In its linear setting the question of the deformation boundary conditions was investigated by the abstractor [*Prikl. Mat. Mekh.* **22**, 2, 238–244, 1958].

K. F. Chernykh
Courtesy Referativnyi Zhurnal, USSR

4668. Mushtari, Kh. M., Some reversed boundary problems in the nonlinear theory of slanting shells of revolution (in Russian), *Izv. Kazansk. Fil. Akad. Nauk SSSR, Ser. Fiz.-Mat. i Tekhn. Nauk* no. 12, 63–67, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12228.

Nonlinear equations for slanting shells of rotation are investigated and two reverse problems are proposed: (1) To find a shell of such a nature that at an assigned pressure it will assume the form of a spherical segment of the assigned curvature; (2) to find a principle for the distribution of the normal pressure which will induce the spherical segment to be transformed in a similar fashion, that is to remain spherical. By virtue of the nonlinearity of the equations the form of the undeformed shell being sought in the first problem and the principle for the distribution of the pressure being sought in the second problem both depend on the bending of the shell. V. I. Feodos'ev
Courtesy Referativnyi Zhurnal, USSR

4669. Mushtari, Kh. M., The mean deflection of a slanting shell, rectangular in plane and resting on ribs flexible in its plane (in Russian), *Izv. Kazansk. Fil. Akad. Nauk SSSR, Ser. Fiz.-Mat. i Tekhn. Nauk* no. 12, 53–62, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12237.

Variational methods, as is known in determinations of stresses in elastic systems, give an accuracy which is appreciably lower than when determining transpositions. In order to calculate the stresses with sufficient accuracy it is essential to introduce a large number of varying parameters into the approximating functions. In the solution of nonlinear problems linked with large curvatures of the shells, this leads to indeterminable calculation difficulties. The author proposes to give as a function for the deflections of a slanting cylindrical panel supported on flexible ribs an expression in the form of a series

$$w = \sum_{m\,n} w_{mn} \cos m x \cos n y$$

The equations for slanting shells are integrated by the Bubnov-Galerkin method. In further computations in the linear portion of the solution the items are retained which contain w_{11}, w_{13}, w_{31}, w_{15}, w_{51}, w_{17} and w_{71}. In the nonlinear portion it is proposed to retain the items containing w_{11}^2 only. The method indicated enables a sufficiently simple and precise determination to be made of the stresses in the panel, provided the panel deflections are not too big. Evaluations are made of the errors which emerge and an example of the calculation is given. V. I. Feodos'ev

Courtesy Referativnyi Zhurnal, USSR

4670. Bolotin, V. V., The stability of a thin-walled spherical shell under the influence of periodic pressure (in Russian), Raschetyi na Prochnost', Vol. 2, Moskva, Mashgiz, 1958, 284–289; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12248.

A clamped spherical shell is under the action of an external pressure varying with time in accordance with the principle

$$p = p_0 + p_t \cos \Theta t$$

Insofar as the small transpositions are investigated the problem, as is usually the case in similar instances, merges with Matier's equation, and regions of instability are established. The question is investigated without a detailed analysis of the amplitude of the vibrations in the case of loss of dynamic stability, the nonlinear equations for a slanting spherical shell being used here.

V. I. Feodos'ev

Courtesy Referativnyi Zhurnal, USSR

4671. Podstrigach, Yu. S., The thermal field in thin shells (in Russian), *Dop. Akad. Nauk URSR* no. 5, 505–507, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12249.

On the assumption of the linear distribution of the temperature over the thickness of the shell $t = p + \gamma q$, where γ is the coordinate along the normal to the mean surface, a system is derived for the case of a nonstationary problem of two differential equations for the magnitudes p and q. Abstractor's note: With the accepted principle for the distribution of the temperature over the thickness of the shell the magnitudes p and q are determinable directly from the assigned boundary conditions on the surface of the shell

$$\frac{\partial t}{\partial \gamma} = -k_1 (t - t_1) \text{ with } \gamma = \frac{\delta}{2}$$

$$\frac{\partial t}{\partial \gamma} = k_2 (t - t_2) \quad \text{with } \gamma = -\frac{\delta}{2}$$

where δ is the thickness of the shell, t_1, t_2 are the temperatures of the medium surrounding the shell, k_1, and k_2 are the coefficients of the relative heat emission. These conditions suffice amply for the determination of magnitudes p and q; the equations themselves, obtained in the paper being reviewed, link up between them the given temperatures, t_1, t_2 and cannot be fulfilled [*See* V. I. Danilovskaya, *Izv. Akad. Nauk SSSR. Otd. Tekhn. Nauk* no. 9, 157–158, 1957]. V. L. Prisekin

Courtesy Referativnyi Zhurnal, USSR

4672. Kraus, H., Bilodeau, G. G., and Langer, B. F., Stresses in thin-walled pressure vessels with ellipsoidal heads, *ASME Trans.* **83** B (*J. Engng. Industry*), 1, 29–42, Feb. 1961.

Authors solve the Love-Meissner equations for the ellipsoidal shell by means of difference equations on a digital computer. Theoretically computed stresses are compared with experimental data and check fairly well. Results are presented for a rather wide range of shell parameters. Readers may also obtain copies of the code through the SHARE program of the IBM Corporation.

F. I. Niordson, Denmark

4673. Mushtari, Kh. M., and Surkin, R. G., Medium bending of shallow spherical panels, square in shape, with nonlinear relations between stresses and strains (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **2**, 162–165, 1960.

Differential equations for moderate deflections of shallow shells (corresponding to Karman's equations for "large deflections" of plates), derived by the first author in the book "Nonlinear theory of elastic shells" [AMR **14**(1961), Rev. 682], are extended to physically nonlinear materials. The relation between the stress intensity σ_i and strain intensity e_i is assumed in the form $\sigma_i = E_0 e_i (1 - \gamma e_i^2)$, where the constant γ is small so that $(\gamma e_i^2)^2$ is negligible compared with unity. Applying Galerkin's method, a solution in the form of double trigonometric series is obtained.

Numerical examples show that, for a given deflection, the stresses in a material with very small nonlinearity may be about 25% lower than in a physically linear material.

A. Kornecki, Israel

4674. Eason, G., The minimum weight design of circular sandwich plates (in English), *ZAMP* **11**, 5, 368–375, Sept. 1960.

A criterion due to Drucker and Shield (Proc. 9th Int. Congr. Appl. Mech. '56) for the optimum design of sandwich plates is applied to circular plates with both simply supported and clamped edge conditions. Cases of central and annular symmetrical loading are considered and the Mises-Hencky yield condition is used. Numerical results are compared with those of previous investigations, which used Tresca's yield condition.

W. S. Hemp, USA

4675. Kacner, A., and Kaczkowski, Z., Application of tabulated functions to the computation of deflections and mechanical quantities of orthotropic plate strips and semi-strips (in Polish), *Rozprawy Inz.* **8**, 4, 873–897, 1960.

The solution of the boundary-value problems of the theory of elasticity is based, in most cases, on infinite functional series. This method, in addition to considerable advantages, has the drawback of making difficult the investigation of the functions of which they are expansions. Many attempts were and are still being made to sum these series. One of the successful ones is the present paper. Starting from the expansion of the function of the complex variable in Taylor series the authors obtain, after appropriate rearrangements, sums of the type

$$\sum_{m=1}^{\infty} \frac{e^{-m\pi\eta}}{m^s} F_1(m\pi\alpha) F_2(m\pi\xi), \sum_{m=1}^{\infty} \frac{e^{-m\pi\eta}}{m^s} \sin m\pi\alpha F_1(m\pi\tau) \times F_2(m\pi\xi),$$

where $s = 1, 2, 3, 4$ and F_1, F_2 are either sin or cos. These sums are expressed in a tabulated form in another paper by the second of the present authors and expressed either by involved elementary functions or improper integrals of these functions. The relations obtained enable the expression in a closed form of the equations determining the deflection surfaces of an orthotropic strip and semistrip, of which the long edges are simply supported and the boundary conditions on the shorter edge are arbitrary and acted on by a concentrated force or moment in any direction.

It is worth observing that a combination of the results of the present paper with the equations of S. Woniowski-Krieger, determining the Green's function for an isosceles triangular plate, would enable us, perhaps, to obtain relations between the equations derived by the present authors and the theta function.

R. Solecki, Poland

4676. Kitover, K. A., Elastic equilibrium of thin plates of orthotropic material (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **30**, 85–98, 1960.

The problem of bending of thin infinite orthotropic plates is investigated. The considered plate is assumed to be in the state of plane stress. Three fundamental types of orthotropy resulting from different elasticity characteristics of the given plate are taken into account.

The problem is solved by means of special functions of complex variable. The solution is supposed to satisfy the load and boundary conditions along the edges of a half-plane and of an infinite strip respectively.

The solution of an infinite plate and of a plate having the form of a half-plane is then obtained in a closed form. In the case of an infinite strip the solution is expressed with aid of a Fourier integral, whose integrand decreases very fast with the independent variable.

No numerical solution is given.

V. Kovarik, Czechoslovakia

4677. Sosnin, O. V., Redistribution of stresses in continuously rotating disks in the first stage of creep (in Russian), *Zb. Prikl. Mekh. Tekh. Fiz.* **2**, 152–156, 1960.

Paper presents a calculation of a solid uniformly heated disk of a constant thickness under the conditions of unsteady and steady creep. The author starts from the hypothesis of strain hardening using basic equations of the theory of creep analogically, such as those employed in the theory of plasticity of Saint-Venant. The relation between the deformation and stress at uniaxial state of stress is selected in the form $\dot{p}\rho^{\alpha} = k \exp(S/A)$ where $\dot{p}$ is the rate of creep, S the maximum shear stress, α, k, A, constants depending upon the temperature and material used. The author further supposes that at any given time interval the disk is divided into two regions, namely (1) region $0 < r < r_{1,2}$ where $\sigma_r = \sigma_\theta$ (σ_r, σ_θ are radial and circumferential stress, resp) and (2) region $r_{1,2} < r < r_a$ where $\sigma_\theta > \sigma_r > 0$ (r_a are the disk o.r.). On the radius of contact $r_{1,2}$ of these two regions the equality of radial and circumferential stresses and circumferential deformations holds true. The calculation can be carried out by numerical integration. The case of steady creep, when $\sigma_r = \sigma_\theta$ applies to the whole disk, is also treated. A numerical example is presented.

J. Valenta, Czechoslovakia

Buckling

(See also Revs. 4652, 4696)

4678. Kloppel, K., and Protte, W., The tipping of circular, curved bars (in German), *Stahlbau* **30**, 1, 1–15, Jan. 1961.

Paper concerns lateral buckling of a beam in the form of a circular arch, with narrow rectangular or I section, subjected to outward or inward thrust acting on its ends. Stiffnesses in bending, torsion and bending-torsion (torsion with variable twist) are taken in account according to the theory of AMR **8**(1955), Rev. 1974 and *Ann. Ponts Chaus.* **124**, 3, 255–374, May 1954. The differential system is reduced to difference equations in matrix form. Numerical calculations are carried out by IBM-650 plant. For arch angles of 90°, 135° and 180°, the results are given in graphs in terms of nondimensional parameters. More general data including the effect of arch angle are given in the case of negligible bending-torsion stiffness. Experimental data show satisfactory agreement with theory.

P. Cicala, Italy

4679. Bresler, B., Design criteria for reinforced columns under axial load and biaxial bending, *J. Amer. Concr. Inst.* **32**, 5, 481–490, Nov. 1960.

Several design criteria for columns subjected to compression combined with biaxial bending are discussed. The approximate load-carrying capacity is defined in terms of easily determined parameters without the cumbersome trial and error procedures.

From author's summary

4680. Austin, W. J., Strength and design of metal beam-columns, *Proc. Amer. Soc. Civ. Engrs.* **87**, ST 4 (*J. Struct. Div.*), 1–32, Apr. 1961.

This paper contains a review of the state of knowledge relating to the behavior, strength, and design of metal members subjected to combined axial compression and bending as of 1961. Practical procedures are presented for the prediction of the strength of individual members under known loading and supporting conditions, and then, these procedures extended, when possible, to apply to assemblages of members. Recent developments which should make possible a more accurate analysis and a more economical design are described. Although primarily a review of published work, this paper also contains some original interpretations and some re-casting of previously published work to give a unified treatment to the subject.

From author's summary

4681. Dow, J., How to determine critical buckling loads for columns loaded between supports, *Mach. Design* **33**, 4, 167–168, Feb. 1961.

4682. Sawyer, D. A., Various instability modes of the fixed base column, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 7 (*J. Struct. Div.*), 61–79, July 1960.

Paper gives "classical" analysis of the critical buckling of a column loaded under a variety of conditions. The base is taken to be fixed, and the top to be loaded through a frictionless roller, or through a pinned link. The analysis has application to a lift-slab operation.

J. Heyman, England

4683. Clark, J. W., and Hill, H. N., Lateral buckling of beams, *Proc. Amer. Soc. Civ. Engrs.* **86**, ST 7 (*J. Struct. Div.*), 175–196, July 1960.

Paper gives a brief summary of the information dealing with elastic buckling of beams and girders. Based on elastic potential energy, a general equation for computing the lateral buckling strength of beams is developed. However, it is claimed that the derived equation may be used to predict inelastic buckling strength with the aid of minimal experimental data. A number of simplified formulas have been developed for designing beams of various cross sections. Experimental results are cited.

L.-W. Hu, USA

4684. Zyczkowski, M., The influence of weight on creep buckling of bars (in Polish), *Rozprawy Inz.* **8**, 3, 511–528, 1960.

By the term of creep buckling (which may raise some objections) author understands the compression of an initially curved bar, with a force acting on the end and the weight. The material is subjected to the law of linear creep, the elastic deformation being also accounted for (the Maxwell law).

The fundamental fourth-order differential equation is established for the above problem in dimensionless variables. The author seeks an initial deflection line such that the ordinates grow proportionally with time. Using the perturbation method he finds an equation for reduced loading force, with which the bar undergoes the same buckling rate as with the real force together with the weight. Then he determines the equation of the initial deflection line. It is found that this line differs very little from the sine surve and that the influence of the weight may in this case (both for creep buckling and elastic buckling) be accounted for by introducing a concentrated force acting on the end and equal to 0.3 the weight.

The paper is intended for scientists and designers.

J. Mutermilch, Poland

4685. Hunyady, F., Lateral buckling of steel girders with variable cross section (in Hungarian), *Mélyépítéstudományi Szemle* **10**, 3, 138–144, Mar. 1960.

Author presents approximate solution of lateral buckling of steel girders with nonuniform cross section. Method is based on the introduction of an "equivalent" girder with uniform cross section having the same critical load as that with variable cross section. Numerical example is presented for the case of a beam with nonuniform I section. O. Halasz, Hungary

4686. Czulak, J., Computation of the stability of combined cylindrical and conoidal gridwork coverings (in Polish), *Rozprawy Inz.* **8**, 3, 531–563, 1960.

This is a development of the author's method of analysis of the stability of space roof gridworks. Critical load is obtained for several cases of rectangular gridwork composed of arches and beams constituting a cylindrical or conoidal surface. Cases of gridworks with three or four arches are considered. The load is uniformly distributed over the roof. It is assumed that continuous three- or four-span beams are supported by the arches in a certain idealized manner. The beams transfer only the vertical reactions acting in the planes of the arches. It is assumed that the arches may buckle in their planes only.

Cooperation between all the beams and arches is taken into consideration. Antisymmetric buckling is assumed in relation to the longitudinal axis of the gridwork or in relation to both the longitudinal and the transverse axis simultaneously. The arches do not buckle out of their planes because they are stiffened by the beams and the buckling of beams does not take place in a conoidal gridwork. Before buckling the arches are compressed axially because they have a parabolic form and the load is uniform. Bending moments appear as a consequence of buckling.

The critical load is obtained by means of the differential equation of the deformed axis of the arch. The differential equation is replaced by a difference equation and represented in the form of a homogeneous system of algebraic equations, in which the unknowns are the displacements of the deformed gridwork. The coefficients of these equations depend on the form and the elastic properties of the gridwork and on the roof load. Since the buckling deflections are different from zero the determinant of the coefficients of these equations must be zero. From this condition critical loads are obtained.

Numerical examples show that a cylindrical gridwork is more stable than the corresponding conoidal one. The results are represented in the form of simple formulas and tables; therefore they may be used by designers of roof structures of materials working in the elastic range. A. Borcz, Poland

4687. Baldacci, R., Considerations on the stability of elastic equilibrium (in Italian), Centro Studi Constr. Metall. Univ. Pisa no. 5, 7 pp., 1959.

After discussing concept of indifference of elastic equilibrium, author shows that classical elasticity theory fails in solving problems of stability. He derives new general equations containing additional terms representing effect of deformations on state of stresses. Author claims that these equations are completely rational and that usual treatment of stability problems is unsatisfactory from theoretical point of view. Finally, method is applied to case of Euler column, derivation of buckling load being, of course, more complicated than usual.

F. J. Plantema, Holland

4688. Pogorelov, A. V., Supercritical deformation of compressed cylindrical shells, *Soviet Phys.–Doklady* **5**, 5, 1099–1100, Mar./Apr. 1961. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **134**, 1, 62–63, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4689. Darevskii, V. M., and Kshnyakin, R. K., Stability of a cylindrical shell reinforced by a ring under the action of external pressure, *Soviet Phys.–Doklady* **5**, 5, 1101–1104, Mar./Apr. 1961. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **134**, 3, 548–551, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.

4690. Hess, T. E., Stability of orthotropic cylindrical shells under combined loading, *ARS J.* **31**, 2, 237–246, Feb. 1961.

The increasing use of composite reinforced materials in structures requires the stability analysis of orthotropic shells. Appropriate equilibrium equations are presented for orthotropic cylinders based upon the use of the simplified Flugge equations. Solutions are obtained for combined axial compression and external pressure for a wide variety of stiffness parameters. The results are presented in a useful series of charts that permit rapid determination of buckling loads. G. Gerard, USA

4691. Troisvallets, R., and Montel, R., Tests concerning the collapse of smooth metal pipes embedded in concrete and subjected to external pressure (in French), *Houille Blanche* **15**, 5, 551–559, Sept.–Oct. 1960.

The problem under consideration is that of the resistance of unreinforced linings to external pressures. Authors describe the large program of tests conducted on sections of concrete embedded in 4100-mm diameter pipe with varying wall thicknesses and with shape defects and clearances between the plating and the concrete. Article deals with the composition of the sections that were tested, with the methods and equipment used for the tests and measurements (measurement of initial shape followed by measurements of stress and strain during the tests), and gives the test results. Analysis of the results brings out the special pattern to which the shape defects conform as well as the effect they have on the localization of the collapse wave and on the limiting pressure. They also confirm the effect of the clearances between the concrete and the plating. These results and comparison with various methods of computation lead to the conclusion that the shape defect factor, which has previously been neglected, should be allowed for when determining the pressure under which linings will collapse. The method of using a template to measure shape defects gives coherent results.

From authors' summary

4692. Montel, R., A semi-empirical formula for determining the limiting external pressure for the collapse of smooth metal pipes embedded in concrete (in French), *Houille Blanche* **15**, 5, 560–568, Sept./Oct. 1960.

Using classical strength-of-materials techniques together with the results of the tests described in a previous article [see preceding review] and the results of earlier tests, author derived a formula for the limiting collapse pressure for unreinforced linings, which allows for shape defects and for the clearance between the concrete and the plating. This formula satisfactorily checks the recent tests on 4100-mm diameter pipes as well as those performed by Borot on 955-mm diameter pipes. It also enables stress-versus-pressure curves to be drawn which check well with those obtained during the tests. Further tests on a 200-mm diameter model verified the results and enabled limits to be a set within which the formula could be used, this being necessary because of its semi-empirical nature.

From author's summary

4693. Tanaka, M., Effect of initial out-of-roundness on the failure pressure of thin-walled cylinder subject to uniform external pressure, *Bull. JSME* **4**, 13, 26–32, Feb. 1961.

To find the effect of initial out-of-roundness on the failure pressure of thin-walled cylinders whose lateral sides were subjected to uniform external pressure, failure experiments on 9

thin-walled cylinders were conducted. The cylinders were of such dimensions that, if they were completely round, the computed values of yielding pressure would be smaller than those of elastic buckling pressure.

Results were as follows: (1) The initial out-of-roundness of the machined cylinders used in the experiment need not be taken into consideration for the calculation of their failure pressures. (2) The magnitudes of initial out-of-roundness of the cylinders can be determined by a method similar to that proposed by Holt.

From author's summary

Vibrations of Solids

(See also Revs. 4569, 4589, 4642, 4652, 4719, 4724, 4793, 4802, 5069, 5108)

Book—4694. Fouille, A., Physics of vibrations [Physique des vibrations], Paris, Dunod, 1960, xvi + 552 pp. (Paperbound)

This is an elementary text for students of applied engineering. It presupposes an acquaintance with ordinary differential equations, but mathematics is held to a minimum, and emphasis is placed on phenomenological explanation and laboratory applications. The book is divided into four parts which deal respectively with the following subjects: general vibratory motion, acoustics, physical optics, and the theory of radiation. The discussion of general vibratory motion (168 pages) deals with linear systems with one and two degrees of freedom, damping, resonance phenomena relaxation oscillations and wave propagation. The section devoted to acoustics (40 pages) contains, in addition to the usual classical material, a chapter on technical applications of ultrasonics. Diffraction phenomena, polarization and spectroscopy are treated in the section on physical optics (147 pages), which is particularly thorough for a book of this type. The discussion of radiation (203 pages) contains well-written explanations of absorption phenomena, incandescent emission, and photometric methods.

L. Caprioli, Italy

4695. Hersch, J., On the fundamental frequency of a vibrating membrane (in French), *ZAMP* **11**, 5, 387–413, Sept. 1960.

Paper contains detailed mathematical discussion of title problem. A technique is introduced by which bounds are set for the frequencies.

R. E. D. Bishop, England

4696. Schmidt, G., Application of nonlinear integral equations in elastodynamics (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **12**, 1, 89–109, 1961.

Paper deals with the transition from the problem of the second-order nonlinear differential equations of the form

$$u'' + \lambda u = \Psi\{u, u', u''; x\},$$

where Ψ can be a polynomial in u, u', u'' whose coefficients are functions of x of the class C_1, to the problem of nonlinear integral equations; periodical solutions with

$$u(2\pi) = u(0), \qquad u'(2\pi) = u'(0)$$

are looked for. First-, second- and third-order approximations are given and the branching equations are discussed both for undamped and damped cases for first-order regions.

Results are applied to lateral buckling vibrations of I beams.

P. P. Teodorescu, Roumania

4697. Budiansky, B., and DiPrima, R. C., Bending vibrations of uniform twisted beams, *J. Math. Phys.* **39**, 4, 237–245, Dec. 1960.

Frequency equations are obtained for pretwisted beam under various end conditions. Displacements in principal directions are assumed represented by trigonometric series. Variation principle is used. Equations involve infinite algebraic sums and frequency appears explicitly. Calculated curves are given for one type of end condition.

R. N. Arnold, Scotland

4698. Lee, H. C., Forced lateral vibration of a uniform cantilever beam with internal and external damping, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 551–556, Sept. 1960.

The steady-state response problem of a uniform beam with a sinusoidal shaking force at the base is studied for the case where the beam material is the general linear substance represented by a model having an additional spring element in parallel with the Maxwell elements. In the analysis, the stress-strain relationship is applied only to the longitudinal strain of the beam, leaving the shear stress-strain relation to be that of a perfectly elastic material. The exact solution with a numerical example is given for one case where the shear and rotatory inertia effects are neglected. This result is compared with the solution obtained by a tabular method. The results of both methods are in excellent agreement.

From author's summary by D. Williams, England

4699. Vorontsov, G. V., The vibrations of beams in a plane of symmetry in which an applied load is acting (in Russian), *Trudi Novocherk. Politekn. In-ta* no. 78/92, 55–66, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12280.

This is an investigation of the deflection-torsion free vibrations of beams of bisymmetrical section loaded in the plane of maximum rigidity by a transverse load and by compressed longitudinal forces. The problem is solved, with account being taken of the influence of inertia of the turns, by the Bubnov-Galerkin method. The systems of differential equations are converted by the introduction of functions of influence, which represent in themselves the deflection and the angle of turn due to a single force and a torsional couple for a beam on a complex elastic base impeding the deflection and the torsion. The procedure adopted for finding the functions of influence is shown by examples, using eccentrically compressed hinge-supported beams.

S. M. Mulin

Courtesy Referativnyi Zhurnal, USSR

4700. Tondl, A., The stability of motion of a rotor with unsymmetrical shaft on an elastically supported mass foundation (in English), *Ing.-Arch.* **29**, 6, 410–418, 1960.

Paper derives expressions for instability regions as a function of shaft speed and ratio of the two shaft stiffnesses relative to the two principal bending axes. Solutions are obtained under the assumption that ratio of shaft stiffnesses is near unity. Curves are plotted showing instability regions for several representative cases with different values of foundation stiffnesses relative to shaft stiffnesses and for different values of damping. In general it appears that an increase in shaft asymmetry increases the instability.

C. D. Pengelley, USA

4701. Dotinko, F. M., On the stability of automatic balancers for dynamic balancing (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 4, 38–45, July/Aug. 1959.

Author investigates stability of operation of a Thearle-type automatic balancer (see E. L. Thearle, "A new type of dynamic balancing machine," *ASME Trans.* **54**, no. 12, 1932) consisting of two balls, each of mass m, that are free to move in a ring mounted on a shaft. For a simply-supported shaft having a single mass M in plane of automatic balancer, author confirms well-known result that a necessary stability condition is operation above critical speed. Author also derives and plots sufficient stability conditions showing effect of mass ratio m/M and damping, and concludes that automatic balancing principle is applicable primarily to light shafts with small M.

Author also investigates flexible shaft with distributed unbalance and with either one or two automatic balancers mounted on shaft in intermediate planes. He states following necessary condition for stability: With a single automatic balancer, shaft rotational speed must be above an odd critical speed of shaft, and below corresponding even critical speed of shaft with an intermediate support at section containing automatic balancer. With two automatic balancers, shaft rotational speed must be above an even critical speed of shaft, and below corresponding odd critical speed of shaft with intermediate supports at sections containing automatic balancers.

Figs. 1a and 1b of paper appear to have been interchanged.

T. P. Goodman, USA

4702. Matveev, V. I., A general method for balancing the rotors of machines in their bearings (in Russian), *Trudi Donetsk. Industr. In-ta* **26**, 97–117, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11299.

The method for the dynamic balancing of rotors as stated in the title is described. A linear relation is assumed to exist between the magnitudes of the vector of the force acting on the bearing and the vector of the vibrations of the bearing; a constant shear for the phase is assumed. Relations are obtained which form the starting point of the calculations, as the result of the solution and the investigation of the differential equations for the motion of a rigid rotor on elastic bearings with friction present proportional to the velocity of the change of positions. The determination of the magnitudes and location of the balancing loads is carried out on the basis of measurements obtained in three trial runs of the rotor: with an initial condition of inequilibrium, with two identical trial loads fastened simultaneously to the two balancing planes, creating a pair of centrifugal forces and, finally, with one of these loads left in one of the planes. In order to furnish proof for the calculation use is made of the relation existing between the amplitudes of the bearings' vibrations, the vector of the unbalanced centrifugal forces and a force in one of the balanced planes which enters into the pair of forces with a momentum equal to the momentum of the unbalanced forces and an arm equal to distance between the balancing planes. Concepts are introduced regarding the conventional machine which enables the dynamic problem of balancing to be solved as a problem in statics. Three interconnected methods of balancing are investigated: successive calibration in each of the balancing planes and simultaneous balancing in two planes, both in the absence and in the presence of the shear phase between the vectors of the amplitudes of the vibrations, produced by fastening a pair of the trial loads or one of them. Recommendations are given in regard to the selection of a rational means of balancing for various machines, and it is shown which of the measurements of the vibrations (in transverse, vertical or axial directions) have to be placed in the forefront of the calculations in order to attain the very best results from the balancing. When so doing the selection of the balancing method is determined not by the measurements of the actual machine but by the measurements on a machine, the lay-out of which permits the direction of the balancing to take the most rational route. Each of the methods of balancing investigated is illustrated by examples of balancing on actual machines.

A. A. Jusarov

Courtesy Referativnyi Zhurnal, USSR

4703. Kostandyan, B. A., Torsion of a shaft with a disk shrunk on it (in Russian), *Izv. Akad. Nauk ArmSSR Ser. Fiz.-Mat. Nauk* **11**, 3, 63–77, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12222.

The problem is solved regarding the torsion of a round cylindrical shaft of finite length, on one end of which a disk made of a different material is shrunk, with an applied load distributed over the whole of the free surface of the shaft and the disk. The region of the axial section is divided into three parts, for each of which a stress function is found satisfying the differential equation and the boundary conditions. A numerical example is examined for which graphs are furnished for a series of sections.

B. N. Lopovok

Courtesy Referativnyi Zhurnal, USSR

4704. Poznyak, E. L., The influence exercised by resistance forces on the strength of rotating shafts (in Russian), Probl. Prochnosti v Mashinostr., no. 1, Moskva, Akad. Nauk SSSR, 1958, 3–24; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12572.

The strength is investigated of a small rotating balanced shaft with a disk set in the center of the shaft, when working at a number of revolutions in excess of the critical and when both external and internal friction are in evidence. The forces of external resistance to the rotation are introduced relative to immovable systems of coordinates, when it is assumed that they are in arbitrary relationship to the frequency. As a special case an investigation is made of friction on the hypothesis of viscous friction. The forces of internal resistance to the rotation, in contrast to the generally accepted hypothesis regarding the proportionality of their vibration amplitudes, are also adopted in the form of arbitrary dependence on the frequency. The forces of internal resistance are introduced relative to the axes coordinates, the axes and the shaft rotating together. An explanation is given for the stabilization of the velocity of rotation after loss of stability when the critical velocity has been exceeded. The following cases of loss of stability in relation to the forces of internal friction are analyzed: the forces of internal friction are viscous; the forces of internal friction do not depend on the frequency in conformity with the power principle. The work of the shaft when the disk has not been tightly set on it is examined to ascertain the influence of this factor on the stability of the work done. Values are given for the decrements and the increments of the vibrations, obtained on apparatus designed by the author, in relation to the frequency for a shaft rigidly and elastically set on supports.

A. F. Gurov

Courtesy Referativnyi Zhurnal, USSR

4705. Sakharov, I. E., Forced vibrations of a disk on a horizontal shaft of double rigidity, rotating on anisotropic elastic mass supports (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 53–58, May/June 1959.

Author treats title problem by solving two linear second-order differential equations with periodic coefficient with the help of the development of the solutions in power series of small parameter. Two first approximations for the amplitudes as well as the equations for the natural frequencies are obtained. The secondary critical speeds appear as $\omega = \omega_n/n$, where even n are due to double stiffness and gravity and odd n are due to unbalance. Since system is considered being symmetric no gyroscopics are included.

G. V. Tordion, Canada

4706. DiMaggio, F. L., and Silbiger, A., Free extensional torsional vibrations of a prolate spheroidal shell, *J. Acoust. Soc. Amer.* **33**, 1, 56–58, Jan. 1961.

Numerical and graphical nondimensional results are presented for the first eight "torsional" modes of a prolate spheroidal shell. Paper is purely theoretical.

R. E. D. Bishop, England

4707. Bird, J. F., Hart, R. W., and McClure, F. T., Vibrations of thick-walled hollow cylinders: Exact numerical solutions, *J. Acoust. Soc. Amer.* **32**, 11, 1404–1412, Nov. 1960.

Plane-strain natural frequencies up to 25,000 cps are computed for an elastic cylinder of fixed outer diameter and elastic constants as a function of the inner diameter (down to half the outer

diameter) for modes with 0, 1, 2 and 3 azimuthal nodes. Boundary conditions cover following cases: outer cylindrical surface free, supported or clamped; inner cylindrical surface free, supported or containing a gas under pressure. Frequency equation is a 4×4 determinant whose individual terms are linear combinations of Bessel functions. Roots were obtained by a Univac computer. For the gas-filled case the attenuation of sound in the solid was also computed by assuming the solid to have dilational and shear viscosity. Results show that coupling of shear and dilation in the solid is weak except at low frequencies and that for the gas-filled case there is very little coupling between the gas and the solid.

S. H. Crandall, USA

4708. Bird, J. F., Vibrations of thick-walled hollow cylinders: Approximate theory, *J. Acoust. Soc. Amer.* **32,** 11, 1413–1419, Nov. 1960.

Paper treats same problem as paper reviewed above. Coupling between shear and dilation is treated as a perturbation. Arguments of Bessel functions become large so asymptotic approximations can be used. Thickness eigenfrequencies are obtained by hand computation which agree with previous paper within a few per cent except for the lowest modes of very thick shells.

S. H. Crandall, USA

4709. Salvadori, M. G., and Heer, E., Periods of framed buildings for earthquake analysis, *Proc. Amer. Soc. Civ. Engrs.* **86,** ST 12 (*J. Struct. Div.*), 59–71, Dec. 1960.

The shear, bending, rocking, and translational periods of cantilever beams with linearly varying shear and flexural rigidities, and with or without a concentrated mass at their top, are computed and combined to obtain a formula for the approximate evaluation of periods of framed buildings. This formula is shown to check the experimental value of the fundamental period of one well-known building within about 5% to 10% and to cover the complete range of periods corresponding to the actual range of parameter values encountered in practice. It is suggested that a more accurate formula, possibly based on the present results, could be adopted in the codes to improve the reliability of earthquake analysis.

From authors' summary by J. Solvey, Australia

4710. Krynicki, E., The influence of the shear and longitudinal forces and the rotational inertia on the vibration of foundation frames (in Polish), *Arch. Inzyn. Ladowej.* **6,** 3, 271–305, 1960.

Author derives the familiar equations for vibration of a heavy bar, taking into consideration the influence of the shear force, the rotation of the cross section and the longitudinal force. Then he establishes the transformation equations and gives a table (computed by himself) of the values of the function (2.38) contained in these equations. Next, using known methods he establishes the equations for the computation of the vibration of gate frames of machine foundations. In conclusion, the author gives examples illustrating the influence of the static shear and longitudinal forces on the natural vibration of frames.

The practical value of the paper would be considerably higher if, in the solution of the examples, the author expressed his opinion on the approximate equations used for design problems and cited, for instance, in the works of E. Reusch "Maschinenfundamente und andere dynamische Bauaufgabe," Vols. I–II, 1943, Vol. III, 1942, (Berlin) and I. Kisiel, "Dynamika fundamentów pod maszyny," 1957, (Warszawa). At any rate the present work may be used for theoretical verification of these equations.

The work is intended chiefly for scientific workers, although the results may be applied, with a certain amount of computation work, to engineering problems.

I. Kisiel, Poland

4711. Veinblat, B. M., The damping of vibrations in ferroconcrete bridges (in Russian), *Avtomob. Dorogi* no. 10, 20–22, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12525.

A description is given of the results of investigations on the decrement of damping in ferroconcrete bridges, linked with the internal and external resistances to the vibrations. Based on the experiments the following deductions are made: the decrements of damping decrease with increase in the span of the bridge; the decrements of damping of prestressed spanned constructions are considerably less than in constructions built with ordinary ferroconcrete.

Kh. M. Lokshin

Courtesy Referativnyi Zhurnal, USSR

4712. Ordanovich, A. E., and Strelkov, S. P., An electronic model of the deflection-torsion vibrations of the wing of an airplane (in Russian), *Nauch. Dokladi Vyssh. Shkoly. Fiz.-Mat. Nauk* no. 2, 121–188, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11563.

A description is given of an electronic model designed for use in the study of the deflection-torsion vibrations of a straight wing of high aspect ratio. The model consists of 80 operational parts of which 32 are integrators and 48 summators and phase-inverters. The range of working frequencies is from 20 Hertz to 5 Hertz. For the purpose of deriving the equations for motion the wing is assumed in the form of a discrete mechanical model consisting of eight cells. A structural plan is given of the electronic model which corresponds with the equations describing the motions of the system under investigation. The results of the investigations, using the electronic model, on the natural frequencies and form of vibrations on several wings are described. Comparison of the experiment with the calculation showed that modeling enables natural frequencies to be obtained for the lowest tones with an error not exceeding 2–3%.

M. D. Klyachko

Courtesy Referativnyi Zhurnal, USSR

4713. Worley, W. J., Mechanical vibration filters with nonlinear characteristics, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 369–376, Nov. 1960.

Paper makes differential analyzer studies of isolating characteristics of a vertical suspension system with damping and spring characteristics of the respective forms $|\dot{x}|^{n-1}\dot{x}$, x or $\tanh(kx)$. Specific values of n used were 0.5, 1, 2, and 3. The special case $n = 0.5$, $\dot{x} < 0$; $n = 2$, $\dot{x} > 0$ and that with the inequalities reversed were also studied. The forcing function was sinusoidal. Constants were selected to enable suitable comparisons with the Jacobsen concept of equivalent viscous damping. For the linear spring, $n = 0.5$ gave the smallest steady-state relative displacements and velocities; $n = 3$, the largest. This was also true for the hyperbolic spring except at the largest excitation amplitude. In the latter case $n = 0.5$ ($\dot{x} < 0$); $n = 2$ ($\dot{x} > 0$) was minimum. The major effect of the nonlinear spring was to give asymmetrical characteristics. For the steady-state vibrations the Jacobsen equivalent viscous damping is reliable for the damping form studied. In general (steady-state and transient conditions) $n = 3$ or $n = 2$ ($\dot{x} < 0$); $n = 0.5$ ($\dot{x} > 0$) gave the best isolation.

M. G. Scherberg, USA

4714. Novoselov, V. S., Influence of variable mass of the spool and of the tension of the thread on vibrations of the spindle (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 42–48, May/June 1959.

Author examines theoretically the forced undamped vibrations of the spindle due to unbalance, variable mass of the conical bobbin and periodic thread tension displacement during spooling. First, the spindle is considered rigid. The differential equation is linear, of second order, with variable coefficients. Solutions of

first-order approximation are found. The limits of the two critical speeds are given. It is interesting to note that in resonance the amplitudes are built up with third power of time. Second, the spindle is considered elastic and divided in n portions for the mathematical treatment. The results of this part are less transparent but the conclusion is that the behavior of an elastic spindle is the same in character as a rigid one. A schematic drawing of the analyzed spindle would be desirable.

G. V. Tordion, Canada

4715. Vol'pert, E. G., Calculations for a vibration device with an air spring (in Russian), *Vestn. Mosk. In-ta Ser. Mat. Mekh., Astron., Fiz., Khimii* no. 2, 41–46, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12592.

A calculation is given of a piston-actuated pneumatic device, the spring effect being produced by means of compressed air. A differential equation is derived for the motion of the piston, which is solved by resolution into a series; a case is examined where two or three terms of the series are left over. The results of the experimental investigation carried out by the author are given as well as comparison with the calculation. E. V. Gerts

Courtesy Referativnyi Zhurnal, USSR

4716. Strakhov, G., Characteristics of damping in a two-leaf spring (in Russian), *Izv. Akad. Nauk LatvSSR* no. 9, 117–124, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12593.

The magnitude of the decrement of damping of the vibrating structure is determined by the sum total effect of friction in the joints of the union and the friction in the metal itself. The more significant of the two is the friction in the joints. The simplest problem of all is investigated, the determination of the decrement of damping in the spring, consisting of two leaves connected at the ends by a friction link. A hysteresis loop is drawn and a determination is made of the dissipation of energy for a single cycle in relation to the springs' characteristics to the link's union. The experiments with a model are furnished which confirm the correctness of the calculation procedure. V. I. Feodos'ev

Courtesy Referativnyi Zhurnal, USSR

Wave Motion and Impact in Solids

(See also Revs. 4630, 4793, 4794)

4717. Fredricks, R. W., Diffraction of an elastic pulse in a loaded half-space, *J. Acoust. Soc. Amer.* **33,** 1, 17–22, Jan. 1961.

Author treats diffraction of compressional pulse in semi-infinite isotropic elastic medium. At the surface, the normal displacement is required to vanish over half the plane. Otherwise the surface is free. Wiener-Hopf method is used (not by name) to find amplitude of diffracted pulse. The results are left in integral form for subsequent numerical analysis.

The present results are motivated in terms of geoelastic applications. With slight modification, one might use them to calculate diffraction from a floating object in water.

R. Lyon, USA

4718. Usami, T., Elastic waves generated from an elliptic cylindrical origin under the ground, *Geophys. Mag. Tokyo* **30,** 1, 19–91, July 1960.

Author discusses the elastic waves, specially of shock type, generated from an elliptic cylindrical origin at a depth. For this the surface displacements are divided in three parts, viz, the direct P, the Rayleigh and the surface S waves; for the dilatational origin, and the direct S, the surface P and the Rayleigh waves for the distortional case. Numerical calculations of surface displacements due to both types of origin are made for several cases taking different values for the ellipticity and the inclination of the axis of the origin ellipse. For numerical calculation of the surface displacements due to different types of waves in the aperiodic case the method of steepest descent is used. Calculations are also made for large distances compared with the wavelengths and the results obtained are represented graphically.

A. M. Sen Gupta, India

4719. Mitra, M., Propagation of elastic waves in an infinite plate of cylindrically aelotropic material (in English), *ZAMP* **10,** 6, 579–583, 1959.

Free oscillations of an infinite plate of cylindrical aelotropy are considered for motions symmetrical about an axis perpendicular to the plate. The frequency equations obtained were used to calculate numerically the dependence of wave velocity on wavelength of progressive wave for the first two modes of propagation only. From author's summary by R. Lyon, USA

4720. Meitzler, A. H., Mode coupling occurring in the propagation of elastic pulses in wires, *J. Acoust. Soc. Amer.* **33,** 4, 436–445, Apr. 1961.

The propagation of elastic pulses in wires of circular cross section has been studied for conditions in which pulses having carrier frequencies ranging from 0.5 to 4.0 Mc were transmitted in wires having diameters between 0.1 and 0.2 cm. The pulses used in the experiments were shaped to have relatively narrow frequency spectra. At certain frequencies, called critical frequencies, pulses propagating in certain modes were observed to undergo pronounced distortion in which the peak amplitude of an affected pulse was reduced and the duration of the pulse was increased many times its original length. This pulse distortion associated with the presence of critical frequencies is shown to be caused by coupling between two modes of propagation. In agreement with predictions of the general theory of mode coupling, the critical frequencies are frequencies at which two modes of propagation have the same phase velocity.

From author's summary

4721. Goldshtik, M. A., and Loontyev, A. K., The impact of a sphere on a hard surface and its experimental verification (in Russian), *Inzbener. Fiz. Zh.* **3,** 11, 83–88, Nov. 1960.

Theoretical part of the paper is merely a reproduction of the well-known elementary solution of the motion of a sphere after oblique impact onto a slab (e.g. Geiger-Scheel: Handuch der Physik, **VI**). Experiments made with sugar balls (diameter 3 mm) dropped onto a glass plate proved the validity of the Coriolis theorem concerning the "center of percussion."

It is to be noted that the theory used holds only if the sphere slips on the slab and further that the classical assumption that the sign of the slip cannot change during impact is wrong. This was theoretically and experimentally proved in a number of papers published by Horák and Krupka during the last decade (for references see 9th Congress of Appl. Mech., Abstracts II, 139 a, Brussels 1956 or *Czechoslovak Journal of Physics*, B **11,** 46–65, 1961). Z. Horak, Czechoslovakia

4722. Myachkin, V. I., and Solov'eva, R. N., Study of the propagation of elastic waves of ultrasonic frequency over small distances in rocks, under conditions of natural deposition, *Bull. Acad. Sci., USSR, Geophys. Series* no. 1, 37–44, Sept. 1960. (Translation of *Izv. Akad. Nauk SSSR, Ser. Geofiz.* no. 1, 63–73, Jan. 1960 by Amer. Geophys. Union, Washington 5, D. C.)

A method is presented for studying the propagation of elastic waves with frequencies of the order of 50 kc, at short distances. Results of studies in the Kalush potassium mine are described where the velocities and elastic constants in the potassium salts are determined. From authors' summary

4723. Baker, W. E., Modeling of large transient elastic and plastic deformations of structures subjected to blast loading, *ASME Trans.* **82** E (*J. Appl. Mech.*), 3, 521-527, Sept. 1960.

Author starts with the accepted idea that, at distances from exploding masses proportional to the linear dimensions of the masses, the pressures are equal but the time scale varies directly with those dimensions. Also similar structures scaled to the linear dimensions of the exploding masses and placed at corresponding distances will have identical stresses for small elastic displacements. Main point of paper is to show that what is accepted as true for small elastic displacements is also true for large elastic displacements and for plastic displacements. This is accomplished by examining the governing differential equations. Experiments on slender cantilever beams carried out by the author confirm his conclusions.

Paper represents a significant step forward in the experimental determination of the response of structures to blast waves.

D. Williams, England

4724. Forsyth, E. M., and Warburton, G. B., Transient vibration of rectangular plates, *J. Mech. Engng. Sci,* **2,** 4, 325-330, Dec. 1960.

The response of rectangular cantilevered steel plates to impulsive loads is determined experimentally and analytically. The impulses were obtained in the experiments by dropping a steel ball. In the analyses the five lowest vibration modes were taken into account. Good agreement was obtained.

M. Botman, USA

4725. Hunter, S. C., Tentative equations for the propagation of stress, strain and temperature fields in viscoelastic solids, *J. Mech. Phys. Solids* **9,** 1, 39-51, Feb. 1961.

Author considers a nonisothermal body joining the equation (H) of heat to the equation (V) of isothermal viscoelasticity (with some improvements for taking into account the variability of temperature). He obtains the equations for the propagation of stress, strain, temperature in a viscoelastic solid. If the rate of energy dissipation is neglected, the equations become linear and author may study the propagation of harmonical dilatation waves, in particular the effects on the attenuations of coupling terms between (H) and (V). D. Graffi, Italy

4726. Soper, W. G., Dynamic similitude for lead, *ASME Trans.* **83** E (*J. Appl. Mech.*), 1, 132-133 (Brief Notes), Mar. 1961.

The use of lead as the arresting material in many shock-testing devices requires an understanding of the behavior of lead under dynamic conditions. The size and shape of lead block and the impact velocity of the drop vehicle must be determined to provide the desired acceleration-time pulse on the vehicle. Unfortunately, any mathematical approach to this problem is hampered by two factors. First, the stress law for lead, or the relationship between the stress and strain functions, their time derivatives, and time is not known. Furthermore, the geometry of the arresting block often is not sufficiently simple to allow a mathematical treatment.

This note describes a practical solution to this problem through the use of scale models. Experiments show that, in spite of the complexity of the behavior of lead, its performance in shock tests can be accurately predicted from small-scale tests by a very simple scaling law. Thus it is possible to select the dimensions of full-scale test blocks from trials on blocks of small and convenient size. From author's summary

4727. Lozgachev, V. I., The determination of solid angles, *Soviet Phys.-Tech. Phys.* **5,** 9, 1039-1044, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 9, 1109-1114, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

An expression was obtained for a solid angle for a point source relative to a circular target, consisting of total elliptical integrals, the parameters of which are the radius of the target and its distance from the source. On the basis of the properties of the obtained expression a method is given for the approximate calculation of integral solid angles for plane objects of an arbitrary form, placed in parallel planes. A table is given of integral solid angles for a target and source in the form of coaxially placed disks of various diameters. From author's summary

Soil Mechanics: Fundamental

(See also Revs. 4613, 4740, 4808)

4728. Balla, A., The soil pressure on a sustaining wall (in French), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **29,** 1/2, 99-115, 1960.

Using Jáky's general solution of Kötter's equation for soil with $\phi = 0$, and assuming a linear variation with depth of angle of intersection of slip circles and back of rough wall, author obtains a rigorous theoretical solution for pressures from a completely plastic heavy backfill. This reveals that the locus of axes of the cylinders of slip is, in general, a cylinder itself. A numerical example is given of a wall 10 meters high backed with level fill having $C = 3.5\ t/m^2$ and $\gamma = 1.985\ t/m^3$ giving an horizontal pressure of 53.9 t/m.

Reviewer notes inherent assumption that Mohr envelope of failure extends indefinitely into the tension zone.

T. A. Farrent, Australia

4729. Hoskin, B. C., and Lee, E. H., Flexible surfaces on viscoelastic subgrades, *Proc. Amer. Soc. Civ. Engrs.* **85,** EM 4 (*J. Engng. Mech. Div.*), 11-30, Oct. 1959.

Viscoelastic theory of stress and deformation of a subgrade covered by a flexible plate is presented. It is a help in understanding of problems including time-dependent response. The solutions are deduced by Laplace transformation. Reactive pressure, deflection profile and bending moment of a plate on Maxwell foundation developed by a load distributed uniformly over a circular region and applied suddenly and maintained constant are given.

T. Mogami, Japan

4730. Diagrams used in soil-mechanics (in English), *LGM Mededelingen* **5,** 3, 103 pp., Jan. 1961.

The diagrams are related to density, porosity, water content and degree of saturation. The relation between them are given in formulas from which diagrams are developed. The diagrams are useful when numerous soil samples have to be tested.

From summary

4731. Stork, J., Problems of determination of the degree of dispersion of clay materials (in Czech.), *Stavebnicky Casopis* **9,** 1, 40-48, 1961.

The verification of methods of quantitative determination of state of dispersion of clay materials shows that these methods have to be considered as suitable for practical engineering purposes but not satisfactory from a scientific point of view. From

tests made by the author as well as other investigators a complete lack of consistency of effects of various dispersing agents is shown, e. g. Na_2CO_3, $(COONH_4)_2$, H_2O, $Na_4P_4O_{12}$. The discrepancies of results obtained after different time of wetting in dispersing agent as well as different results of sedimentation are to be explained by entirely differing grain shape and by successive wetting of interlaminary surfaces along which individual particles are clustered together in agglomerations. The electronic microscopy showed that Stokes's law does not hold for clay materials, because the form of elementary particles of clay materials differs fundamentally from the globular form assumed by Stokes's law of sedimentation. From this point of view there is no reason for use of the same granulometric terms for cohesive and cohesionless soils as usual in engineering practice today. The results of tests are to be considered as relative values only, so far the tests were made in the same way, and not as absolute values.

From author's summary

4732. Nosova, O. N., The coefficient of water yield in Boussinesq's equation (in Russian), *Izv. Vses. Nauk-i. In-ta Gidrotekhn.* **58**, 213–221, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12128.

Both qualitative and quantitative studies were undertaken for the processes of water yield in soils where the sub-soil water level was being lowered. The author, after drawing attention to the fact that the water yield may be substantially dependent on the dynamics of the capillary zone, proposes the following formula linking the "effective" water yield m with the full water yield a in dependence on the velocity of lowering the level of the water v and on the coefficient filtration k

$$m = a\left(1 - \sqrt[n]{v/k}\right)$$

where in accordance with different data $2 \leqslant n \leqslant 3$. In normal conditions magnitude m differs from magnitude a by approximately 20–30%.

V. M. Shestakov

Courtesy Referativnyi Zhurnal, USSR

Soil Mechanics: Applied

(See also Revs. 5117, 5119)

4733. Asai, S., Measurements of bearing capacity of subgrade by drop impact method (in English), *Soil and Foundation* **1**, 1, 42–52, Apr. 1960.

Using energy methods, and based on elastic action of a weightless soil in supporting a steel plate after the first blow from a falling weight which gives plastic deformation, author derives a relationship between the initial deflection from a subsequent blow (I-value) and the modulus of subgrade reaction (K_{30}-value) as given by 1.25-mm deflection of a 30-cm diam plate. Author asserts $K_{15} = 1.4\ K_{30}$ and assumes $I_{15} = 1.40\ I_{30}$.

Experiments were conducted on sandy and loamy soils and a concrete pavement, using an 18-cm diam plate with a weight of 11.9 kg falling 30 cm. Points plot well with theory once efficiency of blow is established by least squares ($n = 0.417$). A spring-loaded vibration gage on a tripod was used to connect with plate, with 5:1 magnification and paper speed 30 cm per sec.

Reviewer found many misprints, confusion between "strain" and "compression" (5 places), notes that formula [9] is an approximation to, as well as a simplification of, [8], and views the value used for the modulus of compression of the cushioning with doubt, but considers relationship found still good.

T. A. Farrent, Australia

4734. Fischer, K., The rigid elliptical plate on laminated foundation with application to the rectangular plate (in German), *Bautechnik* **36**, 7, 257–260, July 1959.

Author computes the vertical component of displacement in a (horizontal) elastic half-space due to a vertical point load transmitted to the half-space by means of a rigid elliptical plate resting on its surface. He then uses this solution to determine, approximately, the deflection in a *layered* half-space due to a load transmitted by a rigid *rectangular* plate.

H. Deresiewicz, USA

Processing of Metals and Other Materials

(See also Rev. 4635)

4735. Eilon, S., and Avi-Itzhak, B., Case study in the rolling of steel bars, *J. Mech. Engng. Sci.* **2**, 2, 120–128, June 1960.

Authors study the hot rolling process of steel bars from the industrial engineering point of view: how to minimize scrap by applying statistical analysis.

Ratio of bar to billet length is shown to be practically independent of input length, and the distribution of the ratios is approximately normal.

Authors minimize scrap by proper selection of the billet size according to an adjusted mean bar length $\overline{X}$. Their analysis assumes that the standard deviation $\overline{\sigma}$ of the bar length is independent of $\overline{X}$, and that undersize bars, like trimmings, are scrap.

Experiment shows that $\overline{\sigma}$ increases linearly with $\overline{X}$, and therefore the analysis is valid only for small changes in $\overline{X}$.

L. V. Colwell and A. Henkin, USA

4736. Lippmann, H., and Johnson, W., Temperature development based on technological analysis: Fast rolling as an example, *Appl. Scient. Res.* (*A*) **9**, 5, 345–356, 1960.

This paper is a valuable contribution toward the solution of a class of problems of significant practical importance. As an example, in the steel industry the temperature rise in cold rolling of tin-coated strip may impose a limit on the speed at which the strip can be rolled and thus upon the production rate. In this case the limiting rolling temperature is the melting point of tin.

The paper presents an analysis of the mean rise in temperature due to plastic deformation, taking into account the temperature dependence of the yield stress and strain hardening and neglecting heat-conduction effects.

Using the elementary Siebel-Karman theory of rolling, the mean temperature rise in a medium-carbon-steel strip in the roll gap is investigated. This treatment allows for partial slip in the roll gap. Results of numerical examples are given. The authors conclude that although in one instance the temperature rise was a significant 100 °C, its influence upon the forces and stress distribution is negligible. However, the reviewer believes such a rise may be of far more importance in other aspects of the rolling process.

A. H. El-Waziri, USA

Book—4737. Niedzwiedzki, A., Manual of machinability and tool evaluation, Cleveland, Ohio, Huebner Publications, Inc., 1960, x + 96 pp. $6. (Paperbound)

Author has made a commendable effort to bridge the gap between the shop floor and the mass of information which is already available from universities and company laboratories. Much of the success of a venture of this kind depends on the skill with which the author includes and excludes various aspects of the

problem. If the presentation is to be usable, it must be simple and much must be excluded from consideration. Reviewer feels that the most important item among those which author has discarded but should not have discarded is the economic tool life. In days of throw-away tips, automatic tool-changing, etc., there is no ground for a 60-minute tool life. Economic tool life, as its name suggests, is governed by considerations of money which author has not discussed. In fact, checking on an economics-based theory proposed four years ago by reviewer, General Electric engineers found the most economic tool life in a given turret lathe operation was four minutes.

There are several confusing aspects for the serious student of machining. The most important is the use of the same symbol for two different purposes, e.g., n for the tool-life index and rotational speed. The use of capital delta (Δ) in its normal mathematical connotation and as a symbol for chip thickness, in addition to causing confusion raises the question of the readership at which the book is aimed. Author states that Δ was used "in order to have a distinctive symbol that shop men would find easy to recognize and to use." Reviewer is convinced that any reader who could not understand chip thickness without a semi-pictorial ideograph would not follow the mathematical treatment of index laws. Put more generally, reviewer thinks that shop men can be told "what to do," technologists can be told "why," and that the weakness of this book is in trying to cater to both. If a second edition is planned, it would be better to concentrate entirely on the technologist. R. C. Brewer, England

4738. DeBiase, R. G., Determination of cutter trajectories for contoured turbine buckets, *ASME Trans.* **82** B (*J. Engng. Industry*), 3, 200–204, Aug. 1960.

4739. Nara, H. R., Wright, D. K., Jr., and Briggs, C. W., Studies of the design of steel castings and steel weldments as related to methods of their manufacture, *ASME Trans.* **83** B (*J. Engng. Industry*), 1, 73–90, Feb. 1961.

Fracture (Including Fatigue)

(See also Revs. 4625, 4638, 4755, 4759, 4760, 4761, 4764, 4765, 5130)

4740. Zheltov, Yu. P., Formation of irreversible cracks in mining rock (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 6, 52–56, Nov./Dec. 1960.

Paper discusses the problem of the formation of a vertical crack when load is applied to (and removed from) the rock and irreversible process of deformation considered. A general solution of the problem is divided into several phases: (1) formation of the crack at the initial loading of the rock; (2) determination of residual stresses; (3) determination of residual deformations. On the basis of known assumptions, author converts the given problem to that of solving a plane problem of the theory of elasticity. Several numerical examples are presented.

J. Valenta, Czechoslovakia

4741. Broberg, K. B., The propagation of a brittle crack, *Arkiv for Fysik* **18**, 10, 159–192, 1960.

The propagation characteristics of a crack in a brittle, linear elastic material have been investigated. The material is supposed to be homogeneous and isotropic in strain and fracturing properties. The surface energy is supposed to be zero so that the crack tip velocity is constant. The crack propagation is investigated in terms of displacements and stresses in a semi-infinite solid where the plane surface is subjected to a constant pressure acting on an infinite strip, the width of which is symmetrically increasing from zero with a constant velocity and to a pressure outside this strip such that the normal displacement outside this strip is zero.

The shape of the crack is elliptical just as in the nonpropagating case. The velocity of the crack tip equals the velocity of surface (Rayleigh) waves. E. A. Davis, USA

4742. Yoshiki, M., Kanazawa, T., and Itagaki, H., An improved testing method to obtain the critical stress and arresting temperature of brittle fracture propagation, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 1960, 103–106.

A specimen and test precedure are described for initiating and propagating brittle cracks from one side of a static tensile specimen. A notched protrusion on one side of the sample is loaded separately and held at lower temperature than the body of the specimen. The arresting conditions for the crack propagating at high velocity in the uniform-temperature stress field can be appraised and critical stress for arrest of crack can be established. Examples are shown for low-carbon steel plate.

T. J. Dolan, USA

4743. Kurmanov, M. I., Navrotskii, I. V., and Tomenko, Yu. S., Evaluation of cold brittleness of structural sheet steel, *Indust. Lab.* **25**, 11, 1432–1434, Oct. 1960. (Translation of *Zavod. Lab., USSR* **25**, 11, 1370–1372, Nov. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4744. Shevandin, E. M., A method of estimating the tendency of steel to brittle fracture from the appearance of the break, *Indust. Lab.* **25**, 12, 1573–1578, Oct. 1960. (Translation of *Zavod. Lab., USSR* **25**, 12, 1497–1501, Dec. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4745. Terazawa, K., Otani, M., Yoshida, T., and Terai, K., Fracture of notched bar tension specimen, *Technol. Rep. Osaka Univ.* **10**, 432, 763–778, Oct. 1960.

4746. Lozovskaya, V. F., Pashkov, P. O., and Serebryakov, A. V., Velocity of crack propagation in the plastic failure of metals (in Russian), *Zh. Prikl. Mekh. Tech. Fiz.* **1**, 103–109, May/June 1960.

The problem is discussed of the rate of growth of "plastic" cracks in several materials such as copper, soft iron and aluminum alloys. Authors point out that the rate of growth of a crack is fully determined by the descending part of the curve in the $P - t$ diagram (P loading force, t time).

Tests were conducted with flat specimens. The crack was initiated by making a one-sided 5-mm deep notch. During the test, the condition $P = \delta(a - x)\sigma$ [x length of the crack, δ thickness of the specimen, a thickness of the undamaged part of the specimen ($t = 0$), σ mean tensile stress in section $\delta(q - x)$] was satisfied at every time interval. The paper presents diagrams plotting the loading force $P(t)$ versus the length of the crack x. The relation between the rate of growth of the crack and the variation of load is linear for the materials under test. For the undamaged region having a length $(a - x) \geq 1/5\ a_0$ (a_0 over-all thickness of the specimen) the product $\sigma_\beta \cdot \delta$ (σ_β strength of the material) remains constant during the propagation of the crack. The rate of growth of the crack depends strongly on the initial state of the material. In conclusion the authors point out that work expended on the growth of the crack is directly proportional to the strength of the material and indirectly to the rate of propagation of the crack.

J. Valenta, Czechoslovakia

4747. Kawamoto, M., and Seki, M., On the fatigue strength diagram of steel, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 1960, 12–14.

Authors give a simple method of drawing the time-strength diagram, in which the ordinate and the abscissa represent the stress amplitude σ_a and the mean stress σ_m respectively. The fatigue strength lines at various numbers of stress cycles N are assumed to be straight and parallel to each other. The endurance limit line is obtained by connecting the point representing the tension-compression endurance limit and the point representing the breaking strength on final area. Reviewer believes that this method cannot be valid for small number of stress cycles because, for instance, the line representing $\sigma_a + \sigma_m = \sigma_B$ is certainly one of the family of the fatigue strength lines for $N = 1$, and this line is not parallel to the endurance limit line.

In the proposed torsional stress $\tau_a - \tau_m$ diagram, each fatigue strength line becomes a radial straight line converging to an intersecting point of the endurance limit line and the line $\tau_a + \tau_m = \tau_B$, where τ_B means the torsional strength of material.

The laws are based on a limited number of experimental data.

C. Hoschl, Czechoslovakia

4748. Savel'ev, L. I., Fatigue strength under conditions of asymmetric stress cycling (in Russian), *Vestnik Mash.* **40**, 3, 30–31, Mar. 1960.

Author considers the effect of superimposed stress on the strain amplitude in cyclic fatigue of a plain carbon and low alloy steel. Purpose was to verify the previous work of author which demonstrated no effect of the magnitude of the normal stress on the strain amplitude variation. The particular case involved constant normal stresses superimposed on the cyclic stress level. The behavior can be described in terms of the constant stress and strain and variable stress and strain and three material constants.

R. W. Guard, USA

4749. Nikitin, V. I., A method of processing the results of long-time strength tests, *Indust. Lab.* **25**, 12, 1567–1573, Oct. 1960. (Translation of *Zavod. Lab., USSR* **25**, 12, 1492–1496, Dec. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4750. Watanabe, J., Some observations on the effect of inclusions on the fatigue properties of steels, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 1960, 5–8.

4751. Zaima, S., On the directionality and the fatigue strength under reversed bending in a mild steel sheet, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 1960, 9–11.

4752. Isibasi, T., Fatigue strength of notched stainless steel specimens, Proc. Third Japan Congress on Testing Materials, Tokyo, Sept. 15–16, 1959; Japan Soc. Test. Mat., 1960, 24–26.

The fatigue notch factor is found to increase as the theoretical stress concentration factor increases. Non-propagating fatigue cracks were observed in ferritic steel for stress concentration factor greater than 4.

T. J. Dolan, USA

4753. Dishka, D., and Khaimashi, T., Testing for fatigue in fabrics by vibration (in Russian), *Tekstil'n Prom-st'* no. 1, 82–86, 1959; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12815.

A plan is furnished of apparatus for testing fabrics for fatigue (by the Budapest research institute for the textile industry). In conjunction with this plan a theoretical investigation is undertaken of the vibration processes in testing fabrics for fatigue. The results obtained when calibrating the apparatus experimentally are also given. The fatigue strength and the process of fatigue of wool fabrics during repeatedly varying stretching were studied on seven different kinds of wool fabric, three of which were brands of wool flannel, two of velour and two of cloth. The results of testing the fabrics were the obtaining of values for the number of cycles up to rupture, for the stretching at the upper level by a variable load and for a hysteresis curve after a given number of cycles of repeated loading. These results enabled determinations to be made of the change in the coefficients of rigidity and of the coefficients of shock absorption for the fabrics under test, expressed in functions of the number of cycles. In the course of the process of vibration-testing of fabrics it was observed that the residual stretching at the beginning of the test grows rapidly and then gradually dies off. The work due to hysteresis in fabrics under the action of repeated loads drops significantly. The coefficient for shock absorption of the fabrics gradually drops. The number of cycles up to the rupture of the fabric is found to be in logarithmic relationship to the coefficient of asymmetry of the amplitude of elongation and in linear relationship to the upper limit of the variable load.

F. P. Belyankin

Courtesy Referativnyi Zhurnal, USSR

Experimental Stress Analysis

(See Revs. 4635, 4723, 4746, 4748, 4788)

Material Test Techniques

(See also Revs. 4635, 4748, 4779)

4754. Thrower, E. N., Technique for measuring the elastic properties of bitumens, tars and soils under dynamic loading, *J. Sci. Instrum.* **38**, 3, 69–73, Mar. 1961.

A technique is described for the measurement of the components of the complex (dynamic) Young's modulus (E^*) and the complex rigidity modulus (G^*) of bitumens, tars and soils under sinusoidal loading. The equipment is capable of measuring the complex Young's modulus up to about $|E^*| = 5 \times 10^{10}$ *dyn/cm²* (10^6 lb/in²) and the complex rigidity modulus up to

$$|G^*| = 2 \times 10^{10} \text{ dyn/cm}^2 \ (3 \times 10^5 \text{ lb/in}^2)$$

over the frequency range 5 to 500 c/s and at temperatures between −20 and + 30 °C.

To measure the complex Young's modulus, an axial sinusoidally varying force is applied to a cylindrical specimen by a moving-coil vibrator. Piezoelectric gauges are used to convert the force applied to the specimen, and the displacement of its driven end, into voltages, which are measured on a phase-sensitive voltmeter. The complex modulus is derived from a comparison of these voltages with those obtained using a steel calibration spring. For the measurement of the complex rigidity modulus, the material is sheared in the annulus between two cylinders, the inner being driven by the vibrator. The same two gauges now serve to measure the force applied to the inner cylinder, its displacement and their relative phase

From author's summary

4755. Ryabinin, Yu. N., Influence of all-sided hydrostatic pressure on the deformation of metals subjected to tension (in Russian), *Inzh.-Fiz. Zh.* **1**, 12, 90–95, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12669.

Author surveys the results of tests on the fracture of cylindrical test samples under tension by means of high hydrostatic pressure. It is observed that with increase of pressure there is

a sharp reduction in the diameter of the waist when disruption takes place, while the capacity of the material to change its form increases very considerably. It is also demonstrated that the actual deformation over a wide range of pressures proves to be a linear function of the pressure. Comparable graphs are produced to show the relation between the actual deformation at the time of disruption and the magnitude of the hydrostatic pressure of the liquid for test samples made of carbon and chrome-nickel steels which furnish testimony regarding their linear linkage. Similar results are also obtained for duralumin. A different character of the relationship was obtained when testing brass at pressure reaching 28,900 kg/cm^2. Here the result was that starting with a compression of 4000 kg/cm^2 the actual deformation at disruption point tends to attain saturation. The author obtained results of the same nature for steel containing 0.46% of carbon and subjected to heat treatment, and also by working over some of the Bridgeman tests.

M. V. Malyshev

Courtesy Referativnyi Zhurnal, USSR

4756. Zilova, T. K., Palkin, B. A., Petrukhina, N. I., Ryazanov, N. V., and Fridman, Ya. B., Tension tests with differing amounts of elastic energy in reserve (in Russian), *Zavod. Lab.* **25**, 1, 76–82, 1959; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12671.

The paper consists of a study of the influence of elastic energy on the character of the disruption and of the properties of the materials, carried out on a specially built installation DRP-361 for determining the tension, with differing amounts of initial reserves of elastic energy (the isolated system) and with gradually built-up reserves of elastic energy (the nonisolated system). A description of the apparatus is given. The experiments in the case of the isolated system were carried out with test samples of light high-strength alloys. It is proposed to use the minimum initial load P, necessary to effect the destruction of the test sample on the time up to the disruption moment τ, for the characteristic for evaluating the behavior of the material in relation to the pliability of the loading system and the reserve of plastic energy. The higher the value of P and the greater the value of τ, other conditions being equal, the less is the material sensitive to the pliability of the loading system or to the reserve of elastic energy. τ gets smaller with increase in the initial reserve of elastic energy at constant pliability of the loading system. The smaller the pliability of the loading system the greater will be the damping of the load (the curves load-time become steeper). The damping of the load also increases with increase in the plasticity of the material under investigation. Tests on samples of steel of mark ZOXGSNA, following the nonisolated system, showed the influence of the pliability of the loading system on the character of the diagram of tension. The greater the ductility of the loading system the quicker will be the disruption of the diagram, that is the smaller will be the effect of the ductility of the sample and of its energetic capacity on the part under concentrated deformation.

A. V. Efimov

Courtesy Referativnyi Zhurnal, USSR

4757. Myshkin, S. N., A method for determination of the compressive strength of refractories, *Indust Lab.* **25**, 12, 1587–1589, Oct. 1960. (Translation of *Zavod. Lab., USSR* **25**, 12, p. 1510, Dec. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4758. Martincek, G., The application of dynamic method of resonance using short prisms and cubes for testing building materials (in English), *Stavebnicky Casopis* **9**, 3, p. 181, 1961.

The paper discusses possibilities of using short prismatic or cube-like specimens to test building materials.

First the problem of natural-frequencies of longitudinal vibrations of a parallelepiped with free boundary surfaces is discussed, equations for frequencies are derived on the basis of the theory of coupled vibrations of three one-dimensional systems, and relationships expressing natural frequencies of longitudinal vibrations of cubes, prisms with square cross-section having the relation of sides of the cross section 1:2 are given.

The results of the theoretical analysis and experimental measurements suggest how to measure and evaluate the results to determine dynamic moduli of elasticity in case of use of short prisms and cubes as test specimens for resonance dynamic method.

From author's summary

4759. Taira, S., and Honda, K., X-ray investigation of fatigue damage in metallic materials, *Trans. Japan Inst. Metals* **1**, 1, 43–48, July 1960.

In the present paper, authors adopt an x-ray technique to study the fatigue process, the mechanism of fatigue damage, and the amount of fatigue damage by observing the variation of the half-value breadth taken from the x-ray patterns obtained from variously heat-treated carbon steel wires being subjected to stress in rotary bending. The following interesting results are obtained.

(1) The half-value breadth of the x-ray diffraction line is mainly influenced by the variation of micro-stress, and the effect of fragmentation is an influence factor of the second order.

(2) The relation between b/B (ratio of half-value breadth of the diffraction line) - Log n/N (cycle ratio) is expressed by a straight line, and this relation is independent of the magnitude of stress applied.

From authors' summary

4760. Serensen, S. V., and Buglov, E. G., Programming of fatigue tests for statistical loading, *Indust. Lab.* **25**, 11, 1415–1429, Oct. 1960. (Translation of *Zavod. Lab., USSR* **25**, 11, 1352–1357, Nov. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4761. Krueger, G., The problem of extrapolation in prolonged strength tests, *Indust. Lab.* **25**, 11, 1421–1427, Oct. 1960. (Translation of *Zavod. Lab., USSR* **25**, 11, 1358–1365, Nov. 1959 by Instrument Society of America, Pittsburgh 22, Pa.)

4762. Lisitsyn, V. D., Determination of the extent of local deformation by the microhardness (in Russian), *Zavod. Lab.* **24**, 12, 1490–1494, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12681.

The linkage between micro- and macrohardness is established on the basis of the adoption of the power principle of similarity which only holds good with imprints larger than 30 μ (diagonal) if the exponent of the power, or the logarithmic coefficient, is equal to 2. It is established that with imprints less than 30 μ it is essential to select a coefficient $n > 2$ in order to determine the actual microhardness. For ferrite and α-brass the relation was found of the logarithmic coefficient to the intensity of the deformation, on the basis of measurement of the microhardness of the grains in linear tension, which, in its turn enabled the microhardness to be evaluated in accordance with the degree of local deformation.

M. Y. Shashin

Courtesy Referativnyi Zhurnal, USSR

4763. Yoshizawa, T., and Nagao, H., On the relation between the Vickers hardness numbers and the face angles of pyramidal indenters, *Bull. JSME* **3**, 9, 36–41, Feb. 1960.

4764. Gal'perin, E. L., and Terminasov, Yu. S., An x-ray study of the plastic deformation of steel (in Russian), *Trudi Leningr. Inzh.-Ekon. In-ta* no. 23, 68–79, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12651.

The influence is studied, using a steel of mark 55S2, of plastic deformation produced by compression and filing on the changes in the intensity and width of the X-ray interference bands and correspondingly on the blocks and distortion of a crystalline lattice.

The interval for the deformations due to compression varied from 0 to 66.3%, and here a study was also made of the carbide phases other than the α-phase. It was established that plastic deformation of steel leads to a significant extent of breaking down and distortion of the coherent blocks of the α-phase. With deformation at its maximum, dimensions of the blocks were 3×10^{-5} cm. At the same time a disintegration takes place of the carbide blocks the dimensions of which, with deformation at its maximum, are also of the order of 10^{-5} cm. An investigation of the changes of intensity of the interference lines in X-ray photographs shows that the static distortion of the lattice cannot be looked upon as the result of a disorderly transposition of atoms in consequence of thermal motion. The model furnished to show the local distortions of the lattice of deformed metals requires substantial improvement. An attempt is made to establish the connection between the changes of the physical and mechanical properties of a steel in the process of plastic deformation. It was established that the curves for the changes of the dimensions of the blocks in the α-phase and the microhardness in a function of the degree of deformation bear the character of saturation while the elastic distortions of the blocks increase continuously. The last, should the deformations be large, may attain the elastic limit of the steel. The toughening of the deformed steel follows as a consequence of the creation of a submicrononhomogeneous structure due to considerable disintegration of the blocks in the α-phase and to the formation of strongly distorted boundary regions. Elastic distortion of the blocks plays a determined role.

I. A. Razov

Courtesy Referativnyi Zhurnal, USSR

4765. Sokolov, P. S., Determination of the real characteristics of the strength of plastic metals (in Russian) *Zavod. Lab.* **25**, 1, 95–99, 1959; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12644.

In the course of determinations of the real stresses in the waist of a sample under tension, when taking into account the volumetric stressed state, using the methods developed by N. N. Davidenkov and N. I. Spiridonova [*Zav. Lab.* 11, no. 6, 1945] and also P. Bridgeman ["Investigation of large plastic deformation and rupture," Izd-vo in. Lit., 1955] measurements have to be made of the radius of the periphery in which the generatrix of the cylindrical sample is deflected in the meridianal plane. In order to avoid this rather complicated operation the author proposes to substitute for it a simple measurement of the diameter in the central portion of the waist and indicates that after its genesis has begun the mean stress (actual), built up in a function of the relative change in diameter, alters up to the moment of disruption in conformity with the linear principle (in a more complete form than can be obtained as is customary in a function of the constriction of the area of the transverse section). On the basis of these observations the author then proceeds to convert Davidenkov's and Spiridonova's formulas for the purpose of the determination of all three components of the stressed state in the waist, leaving out of them the radius of curvature of the profile of the waist and using only the changes in diameter of its transverse section.

Note by the abstractor: The existence in all materials of a well-defined link between the form of the waist and the relative changes in the diameter of the sample calls for experimental supporting evidence.

N. N. Davidenkov

Courtesy Referativnyi Zhurnal, USSR

4766. Bundy, F. P., Calibration techniques in ultrahigh-pressure apparatus, *ASME Trans.* **83** B (*J. Engng. Industry*), 2, 207–214, May 1961.

Pressure and temperature calibration of ultrahigh-pressure apparatus is described. Pressure calibration up to 30 kilobars has been established accurately. Above this pressures are not known with sufficient absolute accuracy to compare results reliably with theoretical predictions of behavior of matter. The effect of pressure on temperature-sensing devices, like thermocouples, has been measured over temperature intervals of 100 deg C up to pressures of the order of 100 kilobars. The effect on some thermocouples is appreciable.

From author's summary

Properties of Engineering Materials

(See Revs. 4624, 4625, 4626, 4637, 4742, 4744, 4752, 4753, 4764, 4765, 4821, 4823, 4965, 5022, 5023, 5077)

Structures: Simple

(See also Revs. 4573, 4634, 4645, 4647, 4661, 4662, 4666, 4678, 4682, 4683, 4686, 4709)

Book—4767. Guyon, Y., Prestressed concrete, Vols. 1 and 2, New York, John Wiley & Sons, Inc., 1960, xxiii + 559 pp. $13.50; xx + 741 pp. $16.75.

Volume I is a revised edition [see AMR 5(1952), Rev. 1701]. Information concerning some of the new prestressing systems are included.

Volume II deals with statically indeterminate structures. The general theory for elastic analysis is worked out. The concordant cable profiles are discussed fundamentally for continuous beams, and it is shown how to establish the concordant cable by imaginary load method as well as by analytical methods. Nodal points are discussed. A number of examples of design calculations for continuous beams are given.

Continuous structures with nonuniform cables are treated, and the cable profiles are discussed in the different zones. A design theory for prestressed arches is worked out and a special consideration of the moment distribution in framed structures is given. Different methods of introducing temporary joints in the structures are discussed. The design economy based on the preceding elastic analysis is studied in a special chapter including examples.

The latter part of Volume II deals with tests on statically indeterminate systems. Different methods for evaluating the ultimate moments are discussed. A number of tests on different types of prestressed structures are described, including a portal frame, several types of continuous beams and various slabs. In each case an extensive analysis of the special failure types is given. This part of the book should be of the greatest interest for all working with experimental studies of concrete structures in general.

As an introduction to ultimate load design the author studies the moment-curvature relation and its adaptation up to the envelopes of the ultimate moments. Ultimate load design is presented with detailed examples.

The analysis given in this book of the many different problems of complex prestressed structures are of great value. The problems are attacked in an academic way and the discussions are stringent and extensive. This method makes the book unusually long, and the study is laborious, due to the complexity of the problems.

C. J. Bernhardt, Norway

4768. Mikhailov, V. V., Investigation of the resistance to shear of concrete, ferroconcrete and prestressed beams (in Russian), *Trudi Nauk-i In-ta Betona i Zbelezobetona Akad. Str-va i Arkhitekt. SSSR* no. 3, 5–50, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12542.

Author investigates the distribution of the shear stresses in beams when developing plastic deformation in the concrete and opening up of cracks. The results obtained were verified by

tests on laboratory samples of the beams and also on full-size beams. The experiments confirmed the results of the investigation and furnished material for improving the calculations for the transverse strength. It was established that a skew crack resulted from a break-down of the concrete in the zone under tension over the area normal to the reinforcement; the next stage was the extension of the crack in an oblique direction and, when that ended, its location was found to be approximately horizontal at a level corresponding to the lower side of the compressed zone. In the boundary state, in the case where there was good union between the concrete and the whole of the reinforcement, there was either a crushing effect in the compressed zone of concrete or a fracture in the reinforcement. This gave the author a basis for the deduction that, if the whole of the longitudinal prestressed reinforcement is extended to the support and is in effective union with the concrete, there would be no necessity for supplementary reinforcement to resist the shear and that this type of reinforcement can only be brought in in accordance with constructive concepts.

K. S. Zavriev

Courtesy Referativnyi Zhurnal, USSR

4769. Belkin, N. I., The magnitudes of the moduli of deformation for concrete of high grade brands in the deflection of ferroconcrete components subjected to loading of short duration (in Russian), *Trudi Novosib. In-ta Inzb. Zb.-d. Transp.* no. 14, 108–117, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12555.

Data are given for experiments on the deflection of 36 ferroconcrete beams 1.54 m long, 25 cm high and with a width of 12 to 15 cm under loading of short duration. The concrete's grade varied from 400 to 525. The beams had single reinforcement, the precentage of reinforcement being 2.0–3.5. The deflection was measured by means of 10–12 strain gages, 4 recorders and two clinometers. The basic deductions were: (1) the beam's sections in the deformed state remain almost plane; (2) after the appearance of the first cracks the height of the compressed zone practically remains unchanged, (3) the outline of a diagram of the compressed zone is close to the triangular in all stages of the work of the beams, (4) with changes of stresses in the concrete from 0 to 100 kg/cm^2 a rapid drop occurs in the values of the modulus of deformation of the concrete; with further increase in the stresses the modulus of deformation changes within small limits in practical conformity with the linear principle, (5) the change in the rigidity of the beams depends principally on the changes occurring in the modulus of deformation of the concrete.

K. K. Shkerbelis

Courtesy Referativnyi Zhurnal, USSR

4770. Neville, A. M., and Taub, J., Resistance to shear of reinforced concrete beams: Part 4, Behavior of beams with different types of web reinforcement, *J. Amer. Concr. Inst.* **32,** 5, 517–532, Nov. 1960.

4771. Sahlin, S., Structural interaction of walls and floor slabs—effect of deformation of joints on load-carrying capacity of brick masonry walls, light-weight cellular concrete element walls, and over-reinforced concrete frames, *Instn. Byggnadsstat. Medd.* no. 33, 201 pp., 1959.

4772. Meshcheryakov, L. I., Investigation on the elastic distribution of symmetrical and asymmetrical loads between the principal beams in a semi-span continuous metallic spanned structure (Data from examination of bridges) (in Russian), *Sb. Nauchn. Soobshch. Saratovsk. Avtomob.-Dor. In-ta* no. 12, 65–75, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12481.

The deflections and stresses were experimentally determined in the principal beams of a continuous spanned structure with different dispositions of the loads according to the width of the structure, with the object of verifying the coefficients of the transverse location of the loads. It was established that the actual coefficients of the transverse placings were less than the theoretical by 1.2–1.5 times. The influence of the length of the span on the magnitude of the coefficients of the transverse locations is shown. A formula is proposed for the determination of the ordinates of the lines of influence of the deflections, which, in the author's view, enables a more precise evaluation to be made for the coefficients of the transverse placings; the formula is needed for future experimental checks.

E. A. Beilin

Courtesy Referativnyi Zhurnal, USSR

4773. van Langendonck, T., Linear inequalities—Application for linear programming and for statically indeterminate structures in the plastic region (in Portugese), Associacao Brasileira De Cimento Portland, Sao Paulo, 62 pp., 1959.

Linear inequalities and linear programming are presented in an elementary form. Simple linear programming problems of stocking, mixing and production are exemplified.

Plastic collapse of statically indeterminate structures is studied, the influence of normal and shear forces being considered. Plastic collapse is treated as a linear programming problem and application to simple frames is exemplified.

The clearness of the paper gives it good didactical value.

F. Borges, Portugal

4774. Scheibner, J., Calculation of statically determinate and supported annular girders: Parts 1 and 2 (in German), *Maschinenbautechnik* **9,** 5, 255–265, May 1960; **9,** 6, 292–297, June 1960.

Author shows how to analyze annular girders subjected to point loads or to constant vertical distributed loads. The results are tabulated to facilitate computation.

From author's summary by H. Deresiewicz, USA

4775. Absi, E., Study of hyperstatical systems in the elastic plastic region (in French), *Ann. Inst. Tech. Bât Trav. Publics* **14,** 158, 259–265, Feb. 1961.

A great number of studies and tests were carried out with a view to making maximum use of the plastic properties of materials used in both statically determinate and indeterminate structures. The difficulty that might arise derives from the fact that there is as yet no method for resolving statically indeterminate systems directly in the elasto-plastic field. The aim of the present note is to describe a method based on the hypothesis that in one element of the system the relation between the stress S (moment, normal force...) and the deformation D (curve, elongation...) has the form

$$S = a \text{ th } bD$$

where a and b are two characteristic constants of the element considered.

This method makes it possible to resolve a great number of statically indeterminate systems with a good approximation. The present note deals more especially with truss systems and simple bending.

From author's summary

4776. Suddarth, S. K., Determination of member stresses in wood trusses with rigid joints, Purdue Univ., Agric. Exp. Sta. Bull. 714, 15 pp., Feb. 1961.

Stresses in and deflections of members of nail-glued trussed rafters of "W" and king-post designs are analyzed by the slope-deflection method, assuming one hundred per cent rigidity of the joints. The theoretical values are compared with test data carefully obtained with this purpose in mind. Both types of data are in fairly good or even excellent agreement.

While the engineer familiar with the design method employed is immediately aware of the limitations of the procedure used, a less familiar reader is unaware of these limitations, since they were not listed by the author. Thus, the procedure used cannot be adapted without alterations to framed structure with less rigid joints which permit the members to rotate more or less in the joints. In a desire to simplify the computations, the neutral axis was assumed to be along the center line of the beam; form factors for the beams were disregarded; deformation of the wood perpendicular to grain was excluded from consideration. These simplifications were fully justified in this particular study. However, refinements may have to be given consideration in other cases.

The author hopes to reduce the lengthy work involved in computing the member stresses and deflections by the use of automatic computing equipment. The use of computers would indeed be justified and almost a necessity if this design method is to be made applicable to framed structural components with joints having limited and varying amounts of rigidity.

E. G. Stern, USA

4777. Cohn, M. Z., and Petcu, V. A., On the influence of steel percentage on the plastic adaptability of redundant reinforced concrete beams (in English), *Bull. Acad. Polonaise Sci., Ser. Sci. Tech.* **8,** 11/12, 713–721, 1960.

4778. Hombert, H., and Jackle, H., Influence of an elastic foundation on bending moments and bearing forces in an oblique plate (in German), *Bauingenieur* **36,** 1, 19–26, Jan. 1961.

Paper contains an experimental study of ways of minimizing the difference between the values of bearing forces in the corners of the plate in the title.

H. Deresiewicz, USA

4779. Handegord, G. O., and Hutcheon, N. B., Test buildings in building research, *Engng. J. Montreal* **44,** 2, 37–40, Feb. 1961.

4780. Csonka, P., Formulae for controlling stress calculations of calotte shells (in English), *Acta Techn. Acad. Sci. Hungaricae, Budapest* **29,** 3/4, 355–368, 1960.

Formulas are derived for stresses at special points in doubly curved cupola-like shells, which are supported by vertical arches. Corners, edges and points on axes of symmetry are considered. Results can be applied to check the validity of approximate stress analysis. A numerical example is given, based on a cupola in Belgrade.

W. S. Hemp, USA

Structures: Composite

(See also Revs. 4672, 4674, 4711, 4712, 4772, 5156, 5157)

4781. Szidarovszky, J., New deflection theory of suspension bridges (in Hungarian), *Magyar Tud. Akad. Musz. Tud. Oszt. Közl.* **25,** 1/4, 89–143, Jan./Feb./Mar./Apr. 1960.

Author presents a corrected deflection theory of suspension bridges taking into account several effects previously neglected, such as deflections due to shear force, effects due to elongation of the cable and the hangers, to deviation of the hangers from the vertical. Solution is developed for truss-type and plate-girder-type stiffening trusses with nonuniform cross section.

Practical method of solution is developed in form of influence-line analysis and numerical example is presented.

O. Halasz, Hungary

4782. Wittneben, H. J., Analysis of rotationally symmetric containers (in German), *Bautechnik* **37,** 10, 366–379, Oct. 1960.

Article contains many useful formulas and tables which can help designers to quickly find numerical values of stresses in shells in the form of surfaces of revolution loaded symmetrically with respect to their axis but with varying intensity in the axis direction.

M. Hampl, Czechoslovakia

4783. Bonsall, W., Stresses in reinforced concrete shields for nuclear reactors, *Proc. Instn. Civ. Engrs.* **16,** 259–270, July 1960.

The thermal stresses in a slab of reinforced concrete with a temperature distribution dependent only on distance into the slab are analyzed. For a slab which is constrained to stay flat a temperature difference of about 50 °C can be allowed with only a nominal amount of reinforcement to control cracking. If the temperature difference is greater than about 87 °C, then either the steel or the concrete must be stressed beyond the usual working limits.

The economics of the alternatives of allowing a temperature difference of 87 °C in a concrete radiation shield with the attendant necessity for heavy reinforcement or reducing the temperature difference to 50 °C by means of a suitable "thermal shield" are examined in an elementary fashion. It is decided that the thermal shield is probably the cheaper solution when the incident radiation warrants it.

From author's summary by G. Gerard, USA

4784. Babaev, N. N., Coupled vibrations of the hull and deck of a ship (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **28,** 76–81, 1960.

Author considers the case when the lowest natural frequency of the deck is equal or nearly equal to one of the first frequencies of the hull of the ship. The hull is considered as a beam of variable rigidity. Linear damping is taken into account. The reactions from the structure of the deck enter into the equation of motion of the beam. The deck is a two-way slab. The boundaries of the panels are subjected to vertical displacements resulting from the motion of the ship as a whole. Solutions for the deck are given in terms of series of eigenfunctions of corresponding beam problems. Galerkin's method is used to evaluate the coefficients of the expanded solution. The sum of the reactions along the edges of the panels is calculated and substituted into the equation of motion of the hull. Author expands the solution in series of appropriate eigenfunctions of the beam. Neglecting damping the natural frequencies are found as the eigenvalues of a matrix. There is no numerical example in the paper, but reviewer believes the method outlined is advantageous in numerical calculations.

P. Wilde, Poland

4785. Samson, C. H., Jr., and Bergmann, H. W., Analysis of low-aspect-ratio aircraft structures, *J. Aerospace Sci.* **27,** 9, 679–693, 711, Sept. 1960.

Two methods are outlined in this paper for analyzing low-aspect-ratio aircraft structures. Structure is idealized as a grid of flexural members (spars and ribs) and torsion boxes hinged at the intersections of the flexural grid. Arbitrary external loading is accommodated. The "consistent distortion method" uses sets of member deflection and equilibrium equations, boundary conditions, joint equilibrium and slope-compatibility conditions. This forms a large matrix, inversion of which produces deflections, shears and moments in the members. The "transfer matrix method" relates the deflections, rotations, shears and moments in one member to these in adjacent members (field matrix) or adjoining to loads (point matrix). By successive matrix multiplication the values at one boundary can be expressed in those of the opposite side. Inversion of the small resulting matrix gives a solution from which the deflections and internal loads in the members can again be found with the transfer matrices.

Comparison of results of both methods with analog results and plastic model tests are given and compare favorably. Other methods of solution are mentioned but no comparison is made.

J. Van Winssen, Canada

Machine Elements and Machine Design

(See also Revs. 4577, 4700, 4701, 4705, 5071, 5161, 5166)

4786. Niemann, G., and Jarchow, F., Testing spur gear globoidal worm drives (in German), *ZVDI* **103,** 6, 209–221, Feb. 1961.

The various pairs of teeth are first assessed in accordance with the shape of their lines of contact. The effect of contact line position, tooth size, material, lubricating oil and speed on performance loss and load-carrying capacity can be determined by tests. Then a comparison can be made between the spur gear globoidal worm drive and cylindrical worm gears. The mathematical interrelationships established by the tests can be collated in the form of equations, which enable an approximate calculation to be made of the performance loss and load that can be transmitted at maximum temperature.

From authors' summary

4787. Brugger, H., Dedendum load-carrying capacity of straight-toothed spur gears having various shaped teeth (in German), *ZVDI* **103,** 6, 229–236, Feb. 1961.

Results are given of running tests performed on spur gears of module m = 4 and width of tooth face b = 30 mm, on a loaded test bench. It was found possible to increase the dedendum load-carrying capacity by increasing the working angle of pressure and, consequently, the thickness of tooth at the endangered cross section of the dedendum; this, under favorable circumstances, can be up to 40 per cent. When assessing the quality of the noise of the over-all metered acoustic pressure, it was established that the intensity of sound increased with the working angle of pressure. The method of calculation recommended in German industrial standard DIN 3990 for determining σ_v, the stress-strain load, produced a maximum deviation of ±10% for the entire range of the tooth shapes of $\alpha_b = 17^\circ\ 48'$ to $30^\circ\ 56'$.

From author's summary

4788. Wellauer, E. J., and Seireg, A., Bending strength of gear teeth by cantilever-plate theory, *ASME Trans.* **82** B (*J. Engng. Industry*), 3, 213–222, Aug. 1960.

This paper presents a study of the bending strength of gear teeth based on a cantilver plate approach. A semiempircal solution is given for the finite cantilver plate under transverse loads at any location on its surface. The solution, which is based on the principle of superposition and a proposed moment-image method, shows good agreement with results from strain-gage investigations on cantilever plates simulating the gear tooth. The effects of the position of the line load and the variation of load intensity on the stress distribution at the root are discussed.

Reviewer considers this paper a worthwhile contribution to this difficult type of problem. It might, however, be advantageous to point out that, in the case of gears of wide face, torsional wind up of the gear has a very significant effect on the load distribution across the face of the tooth.

E. M'Ewen, England

4789. Gatcombe, E. K., and Prowell, R. W., Rocket motor gear tooth analysis (Hertzian contact stresses and times), *ASME Trans.* **82** B (*J. Engng. Industry*), 3, 223–230, Aug. 1960.

A valuable addition to the field of analytical gear design, based on evaluation of Hertzian contact stresses and times of contact as functions of permissible transmitted loads. The significance of the relationships between contact times and lubricant relaxation times is emphasized and time of contact as a design criterion is indicated. Difficulties of arbitrary assumptions of geometry and other physical factors are intended to stimulate interest in further detailed theoretical analyses.

A gear reduction system of rocket motor turbine-pump is the object of reported analysis; results at five phases of tooth contact are tabulated for respective gear pairs. Two communicated discussions are presented.

Z. W. Dybczak, USA

4790. Ufert, O., Dynamic rotational error measurements on hobbing machines and their effect on the accuracy of large-size gear-wheels generated (in German), *ZVDI* **103,** 6, 240–247, Feb. 1961.

Continuous single-flank contact tests do much to improve the machining accuracy of gear hobbers. In Czechoslovakia the use of magnetic gauge blocks led to the incorporation of correcting devices, which enable the effects of the indexing worm gear's error to be considerably reduced. Moiré fringe tests have enabled Britain to develop correcting devices, incorporating servo motors, which automatically reduce the effects of the worm gearing's cumulative circular pitch error. (The error of faceplate movement has been reduced to less than 4"). Seismic measurements of rotational error, which can be performed to a still higher degree of accuracy by very simple means, have in Germany formed the basis of a further improvement. Even without a correcting device the cumulative circular pitch error was approx 1"; today it is possible by this means to reduce the cyclic error to 0.2". Large-size gear-wheels generated on machines of this type are distinguished by a particularly low noise level.

From author's summary

4791. Brickman, A. D., Designing conical-disc springs, *Mach. Design* **33,** 6, 141–145, Mar. 1961.

A simplified approach for designing springs with specific load-deflection characteristics in a minimum amount of space.

From author's summary

4792. Loubser, R. S., A simple test to predict the increase in permanent set of springs under repeated loading, *So. African Mech. Engr.* **10,** 6, 147–151, Jan. 1961.

Springs which are occasionally subjected to repeated loads inducing high stresses in the blades may fail due to the development of an excessive permanent set. Experiments on decarburized silico-manganese spring blades showed that after the first loading cycle the permanent set increases log-linearly with the number of cycles. Based on this data, an acceptance overload test of three or four load applications is proposed, during which the increase in permanent set is measured. The method of deriving the limiting acceptable set values during such a test is explained.

From author's summary

4793. Fujii, T., and Okada, Y., On the spring constant of bellows type air springs, *Bull. JSME* **3,** 12, 404–410, Nov. 1960.

Air springs are becoming widely used, particularly in vehicles. Paper is concerned with calculation of spring constants for such components. Several configurations are dealt with and experimental verifications are quoted. Treatment seems unusually obscure.

R. E. D. Bishop, England

4794. Ungar, E. E., and Pearsons, K. S., New constant-force spring systems, *Prod. Engng.* **32,** 13, 54–59, Mar. 1961.

Experimental force-deflection curves are given for mountings with numerous arrangements of buckled-levers, rubbers and plastics. Some shock-response curves are included.

R. N. Arnold, Scotland

4795. Newcomb, T. P., Temperatures reached in friction clutch transmissions, *J. Mech. Engng. Sci.* **2,** 4, 273–287, Dec. 1960.

Expressions are given to determine the torque capacity of a clutch and the rate at which heat is dissipated by dry friction during a single clutch engagement, together with equations to enable calculation of the rubbing-path temperatures as well as temperatures at various depths inside the contracting bodies during the slipping period. It is shown how the maximum friction surface temperature varies with the thickness of the pressure plate, and for design purposes a rapid method of calculating temperatures is given when the slipping time is known. Some curves are given showing the agreement between temperatures determined theoretically and measured experimentally inside a clutch facing during continuous slipping.

The temperatures attained after regular repeated clutch engagements are considered when cooling is by convection alone and a method of calculation is outlined when, in addition to convection losses, radiation losses need to be taken into account. A formula is given to enable the area of the rubbing path to be calculated if the temperature is not to exceed a specified value.

In an appendix the temperatures are calculated when the torque linearly decreases with slipping speed and a comparison is made with the case when the torques are assumed to be constant.

From author's summary by M. Maletz, USA

4796. Newcomb, T. P., Determination of the area of friction surfaces of automotive vehicles, *J. Mech. Engng. Sci.* **2,** 4, 312–324, Dec. 1960.

Expressions for the temperature rise in drum and disk brakes are given for both single stops and regular repeated applications when cooling is taken into account. It is shown how these expressions enable the area of the braking path to be calculated if the temperature is not to exceed a specified value. The effect of the dimensions and the thermal properties of the brake components on the rate of temperature rise and on the limiting temperatures reached during a series of applications is discussed. Finally it is shown how random braking can be reduced to equivalent cyclic applications, and the braking area of a small sports car being driven in the Alps is calculated.

From author's summary by H. Blok, Holland

4797. Spehr, E., The Cardan error in a dual-jointed system (in German), *ZVDI* **103,** 6, 247–250, Feb. 1961.

The Cardan error of a dual-jointed system can be calculated on the basis of the error determined on a simple knuckle joint. When the input shaft runs at constant speed an harmonic oscillation overlaps the output speed and its amplitude is eliminated only when the diffraction angles are equal and the so-called resultant phase angle is a right angle.

4798. Faupel, J. H., Some considerations of the mechanics and design limitations of autofrettage, *J. Franklin Inst.* **269,** 6, 474–489, June 1960.

Results reported on heavy-wall steel cylinders prestressed by autofrettage (plastic radial expansion) are discussed. Optimum design is considered relative to flow, section size and properties of material. It is shown that the ability of steel to accept and retain residual stress is a function of the diameter ratio and strength of the material. Design procedures are suggested.

From author's summary by L.-W. Hu, USA

4799. Shaffer, B. W., and Krause, I., Refinement of finite difference calculations in kinematic analysis, *ASME Trans.* **82** B (*J. Engng. Industry*), 4, 377–381, Nov. 1960.

A well-presented paper on application of finite difference equations to analysis of the kinematics of a cam-follower mechanism. Increased accuracy in calculations is obtained by including second central differences, and comparison is made with exact solution. The paper offers good class-room material for professors in machine design.

D. E. Hardenbergh, USA

4800. Artobolevskii, I. I., A class of connecting-rod curves, *Soviet Phys.–Doklady* **5,** 3, 519–521, Nov./Dec. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **132,** 1, 82–84, May 1960 by Amer. Inst. Phys., Inc., New York, N. Y.)

4801. Freudenstein, F., and Sandor, G. N., Synthesis of path-generating mechanisms by means of programmed digital computer, *ASME Trans.* **81** B (*J. Engng. Industry*), 2, 159–168, May 1959.

4802. Kel'zon, A. S., and Mushina, N. I., Study of vibrations of rapid rotors, taking spring tension into account (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 3, 49–52, May/June 1959.

Author presents a nonlinear mathematical treatment of a very particular spindle design used in spinning machinery.

G. V. Tordion, Canada

4803. Yamamoto, M., and Murakami, K., Bending deflection of a crankcase of high speed internal combustion engines, *Bull. JSME* **4,** 13, 141–148, Feb. 1961.

Bending rigidity of a crankcase of high-speed internal-combustion engines is usually smaller in the horizontal-transverse direction of the engine, because of its common hanger-bearing-type construction. For this reason, the elastic vibration appears mainly in the horizontal direction.

In our research the bending deflection of a crankcase was measured with the load applied to the middle point of a crankcase in the horizontal-transverse direction and the crankcase simply supported at both end-bearing centers.

In analyzing the results of bending tests with several models of crankcase, the calculating equations for the bending deflection of a crankcase were induced from "Torsion theory of thin sections" with some modifications.

The results of bending tests on regular engines show that those equations are practically adequate to calculate the deflection of a crankcase.

From authors' summary

4804. Hansen, J. T., How to use readily available specification data to predict dynamic properties of hydraulic motors, *Mach. Design* **33,** 2, 132–136, Jan. 1961.

Rheology

(See also Revs. 4628, 4811, 4978, 5017)

4805. Coleman, B. D., and Noll, W., Recent results in the continuum theory of viscoelastic fluids, AFOSR TN 60-606 (Mellon Institute and Carnegie Inst. of Techn.), 89 pp., May 1960.

Paper includes a series of results obtained in the field of viscoelastic fluids. A relation between the stress tensor and a functional which depends on the history of the Cauchy-Green right tensor for isotropic homogeneous bodies is used. By particularizations, authors obtain the case of incompressible fluids and the case of simple shearing flow, for which the expression of Green tensor with respect to the first two Rivlin-Ericksen tensors

and the values of the extra stress for a positive dissipation of energy are given.

A class of flows is also examined, such as the Poiseuille flow and helical flow. For the case of the Poiseuille flow, authors use constructions of the functional arguments and show that the results are valid for all incompressible simple fluids and particularly for Reiner-Rivlin fluids.

The study is also extended to oscillating motions and particularly to the periodic shearing flow and it is shown that the normal extra stresses oscillate with a frequency that is twice the frequency of oscillation of the velocity field, the displacement field and the shearing stresses.

By correcting the theory of perfect fluids with the aid of the theory of the Newtonian fluids, authors perform the transition in a formal way, i.e., they assume that the memory has an imperfect character, being more active for more recent periods. This comes to the assumption that the functionals describing the stress admit Fréchet differentials of the first order and for more general fluids they have become Fréchet differentials of the nth order.

The presentation is unique due to the theoretical apparatus employed which, although it requires a higher mathematical background, is advantageous in that it allows a wide systematization of the theory of fluids.

The results obtained permit ways of the development of the theory for more general fluids to be foreseen; thus, if linear functionals defined on vector spaces are considered, it is possible to suggest the construction of rheological laws with the aid of Gateaux differentials. M. M. Misicu, Roumania

4806. Meister, R., Marhoeffer, C. J., Sciamanda, R., Cotter, L., and Litovitz, T., Ultrasonic viscoelastic properties of associated liquids, *J. Appl. Phys.* **31,** 5, 854–870, May 1960.

The principal conclusions about the dynamic mechanical behavior of three associated liquids, hexanetroil 1, 2, 6, butanediol 1, 3, and 2-methyl pentanediol 2, 4, are as follows:

(a) shear and volume viscosities are the same order of magnitude and have the same relatively large temperature dependence;

(b) the shear viscosity is related to a relaxation process so that the associated liquids exhibit a shear rigidity of about the same value as polymers (10^{10} d/cm^2);

(c) comparison of the shear and compressional relaxation times showed that the ratio of these times do not differ by more than a factor of 5. K. Gamski, Belgium

4807. Srivastava, A. C., Rotatory oscillation of an infinite plate in non-Newtonian fluids, *Appl. Scient. Res. (A)* **9,** 5, 369–373, 1960.

Author considers a Reiner-Rivlin fluid and expands velocity components in ascending powers of the small amplitude of oscillation, obtaining the first nonzero approximation for each component. Since the convective inertia terms are neglected this procedure is valid near the plate but not outside the boundary layer. Compare work of S. Rosenblat for a Newtonian liquid [AMR **13**(1960, Rev. 2355]. Unfortunately the author focuses attention on his results away from the plate. F. M. Leslie, England

4808. Astrakhan, I. M., and Grigorian, S. S., On the complete system of equations of a compressible visco-plastic fluid, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **23,** 6, 1637–1639, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

For an incompressible fluid the equations of Kasimov and Mirzadzhanzade [AMR 9(1956), Rev. 3598] containing one coefficient of viscosity and one constant of plasticity, together with the equation of continuity, form a complete system. For a compressible fluid an equation of state relating pressure and density is also required. Authors propose use of Liakhov's equation of state, originally developed for three-component water-saturated soils. F. R. N. Nabarro, South Africa

4809. Gurbanov, S. G., Approximate solution of a problem on the circular nonstationary motion of viscous and viscoplastic liquids (in Azerb.) *Izv. Akad. Nauk AzerbSSR Ser. Fiz.-Tekhn. i Khim. Nauk* no. 2, 65–70, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11913.

Results are given for an approximate solution of the problem of the circular nonstationary motion of a viscous liquid in a round tube and on the rotation of a round cylinder in a viscoplastic medium; also on the problem of the impact of a rigid cylinder on a plate, obtained with the aid of the Slezkin-Targ method [*Dokladi Akad. Nauk SSSR* **4,** no. 3, 1946] and with a variant of this method proposed by Shvets [*Prikl. Mat. Mekh.* **13,** no. 13, 1949]. The results obtained were compared with the available precise solutions. The solutions of new-type problems of this nature are not investigated in the paper. There are misprints in [1, 2], the expressions for F_0 and others. S. M. Targ

Courtesy Referativnyi Zhurnal, USSR

4810. Sorokina, G. V., An experimental investigation of the limiting stress of shear and the viscosity of sludges (in Russian), *Sb. Nauk-i. In-ta Osnovanii i Podzemn. Sooruzb. Akad. Str-va i Arkhitekt. SSSR* no. 33, 57–69, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12363.

A description is given of laboratory-scale experimental investigations of the viscoplastic properties of sludges. The limits of applicability of the Shvedov-Bingham principle are established. The values for the viscosity of the sludges being investigated were determined in the range of 10^8–10^7 poises. The character is established of the relation of the gradient of velocity to the shear stress within certain velocity limits. The relation was examined of the limiting shear stress and the plastic viscosity of sludge to its consistency and structural viscosity. Apparatus is described for the determination of the density of the soil with the aid of reactive isotopes and also for determining the velocity of flow of the sludge in varying conditions.

Z. V. Maslova-Pil'gunova

Courtesy Referativnyi Zhurnal, USSR

Hydraulics

(See also Revs. 4844, 4925, 4933, 4941, 4942, 4943, 4944, 4945, 4948, 5116, 5128, 5132)

4811. Ookawa, A., A model theory of liquid, *J. Phys. Soc. Japan* **15,** 12, 2191–2197, Dec. 1960.

As a structural model of liquid, a crystal threaded by fine network of dislocations is proposed. The main assumptions characterizing the model system are the elastic distortion of the mesh material in bulk rather than the local distortion at the core of the constituent dislocations, the frequency modulation for the transversal shear waves due to the anharmonicity of the potential, and the introduction of degenerated nonharmonic shearing translations. With these assumptions the model system is shown to display a kind of first-order phase transition between an imperfect crystal as representative for the liquid phase and a perfect crystal as representative for the solid phase. The order of magnitude of the frequency modulating coefficient, the only adjustable parameter, is estimated under comparison with the experimental data of noble metals.

From author's summary by Z. Horak, Czechoslovakia

Book—4812. Idel'chik, I. E., Reference book on hydraulic losses [Spravochnik po gidravlicheskim soprotivleniiam], Moscow, Gosenergoizdat, 1960, 464 pp. $4.25.

Book is an unusually extensive collection of experimental data concerning friction losses in pipes and other conduits, and local losses in contractions, expansions, diaphragms, outlets, diffusers, bends, elbows, junctions, nozzles, orifices, grids, valves, etc., also around immersed bodies. Friction resistance is limited to the Colebrook-White formula and its substitutes by Al'tshul' and Filonenko. Treatise on local losses supersedes all publications since the classical work "Die Experimental-Hydraulik" by Julius Weisbach, 1855.

All data, based on experimental work in different countries, are presented in form of numerical tables and nomograms, therefore they can be used almost without being translated. Many data are taken from Russian literature and are unknown to our readers. Among them there are many original derivations by the author, some of them questionable, as, e.g., trigonometric law of velocity distribution after obstacles, which seems to be just a convenient practical simplification. Author calls Mach number the momentum correction factor, known as Boussinesq factor (p. 100), while a real Mach number is applied also (p. 109). A list of references is of great value: it contains over 300 Russian, very little known papers, some 50 German, and 40 English and others.

Book can be recommended for hydraulic and sanitary engineers, heat, air-condition, jet propulsion and other mechanical engineers, particularly for those continuing research on hydraulic losses.

S. Kolupaila, USA

4813. Ulbrich, S., Theoretical and experimental determination of the quantity of liquid flowing through piston slide valves (in Hungarian), *Gép* **12**, 11, 423–435, Nov. 1960.

Author derived the equation of discharge through piston slide valves for different positions of the piston and for different valve geometries. Mathematical derivation of the rate of flow based on the equation of motion is given in detail. Investigating the influence of the fluid viscosity author concludes that it may be neglected in the analysis.

Theoretical results were checked by experiments. Experimental data show good agreement with the theory. Comparison between author's results and the generally used empirical equation indicates that the discharge coefficient is a function of the valve geometry and piston position rather than a constant.

A. L. Simon, USA

4814. Furuya, Y., and Sato, T., Pressure recovery efficiency of short conical diffusers and of roughened diffusers, *Bull. JSME* **3**, 12, 437–443, Nov. 1960.

Better efficiency of a 10° diffusor followed by an abrupt expansion compared with a plain conical diffusor of the same length but larger angle is demonstrated. Likewise, roughening of walls of diffusors in zone where separation may occur is shown to decrease loss. Experiments described show that for best efficiency of a diffusor in a limited space, maximum recovery of pressure should be obtained ahead of the point where separation occurs.

D. W. Appel, USA

4815. Plotnikov, N. I., The experimental determination of the uniformity of distribution of air along the length of a tube with holes in it (in Russian), Za Tekhn. Progress. Byul. Novosib. Sovnarkhoza i Oblsovprofa no. 2/3, 38–41, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11608.

A description is given of an experimental plant designed for the purpose of improving the initial data for the calculation and construction of air-distribution systems. The experiments carried out show that the main and its branching should be set strictly horizontally and that the orifices should be situated in the lower part of the tubes. When making calculations for independent air-distributing systems the velocity of the motion of the air in the branches and in the main should be taken as equal to 10 to 15 m/sec, while the velocity of the air leaving the orifices—to 25 m/sec. When designing a combined water-air drainage system the top row of orifices should be directed vertically upwards, the lower row of orifices, vertically downwards. The distances between the orifices should be identical while the orifices themselves should be disposed chess-board fashion. The diameter of the orifices should be equal to 4–5 mm.

L. Ya. Khaskin

Courtesy Referativnyi Zhurnal, USSR

4816. Bernshtein, L. B., Development of tidal energy in the Chinese Peoples Republic (in Russian), *Gidrotekh. Stroit.* **30**, 6, 50–52, June 1960.

In this paper a short and compact report is given on the utilization of tidal energy in China. There are some interesting examples for employing low output tidal power plants adapted to local conditions; among others, utilization of tidal fluctuation of 6 feet, with two basins, using existing dam and gate system. Another interesting application is when driving irrigation pumps by turbine.

A. Verba, Hungary

4817. Simonov-Emel'yanov, Yu. A., The problem of establishing the character of the limits of approximate stability of complex hydraulic pressure systems (in Russian), *Izv. Karel'sk. i Kol'sk. Fil. Akad. Nauk SSSR* no. 4, 54–66, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11847.

Using N. N. Bautin's method a determination is made of the character of the limits of approximate stability of the following complex hydraulic pressure systems: (1) Two surge tanks, serving at the same time as well-cappings, are located successively on the inlet derivation; (2) the surge tanks are located at the end of the inlet and the beginning of the outlet derivation; (3) two converging pressure derivations are joined to one general pipe-conduit, with a surge tank at its end. For a number of numerical values of the parameters of the system the indications were that for all the cases investigated the limits of the region of stability were "dangerous."

G. V. Aronovich

Courtesy Referativnyi Zhurnal, USSR

4818. Sokolovskii, S. V., Application of the G. Ritter-V. Skryl'nikov method for the calculations of monolithic gravitation dams (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Str-vo i Arkhitekt.* no. 1, 181–188, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12489.

Author examines the influence of the torsional forces on the vertical components of monolithic gravitation dams not taken into account in the Ritter-Skryl'nikov method. He bases his case on the assumption that the load is distributed between the horizontal elements: the beams and the vertical cantilevers. The principle of distribution of the load is determined from the condition that the deflections of the horizontal components in the middle and in the corresponding points of the central cantilever should be equal.

A. S. Kozhevnikov

Courtesy Referativnyi Zhurnal, USSR

4819. Meter, D. M., and Bird, R. B., Turbulent Newtonian flow in annuli, *AIChE J.* **7**, 1, 41–45, Mar. 1961.

...Prandtl mixing-length approach is applied to give a friction factor versus Reynolds number expression for annuli; this expression describes tube flow and slit flow as special cases. No new adjustable constants appear in final result other than those determined earlier for tube flow. Final expression is found to predict friction factors within accuracy of existing experimental data. The mixing-length friction-factor expression is thus substantially

more accurate than usual hydraulic-radius procedure and of comparable accuracy to other recent annulus friction-factor treatments.

From authors' summary by J. R. A. Pearson, England

4820. Mikuta, V. I., and Novikov, B. G., Streamline profiles in circular flow (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **3**, 97–101, 1960.

Authors examine the plane flow past submerged bodies of a liquid in uniform rotation about a fixed point (e.g., in a whirling arm installation). Generalization to this case of Blasius integral theorem is obtained and illustrated for Zhukovskii airfoil.

L. Trilling, USA

4821. Rabinovich, E. S., Some problems in hydraulics of molten metals (in Russian), Gidrodinamika Rasplavl. Metallov, Moskva, Akad. Nauk SSSR, 1958, 128–136; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11821.

The basic equations for hydraulics applicable to the motion of molten metals are given. The results of measurements of the profile of velocities in a stream of molten cast iron in a channel with a triangular cross section showed a pure turbulent structure for the flow. The conclusion is reached, on the analysis of the results of experiments with analogs of molten metals—salol, naphtalene and paraffin—that three states can be met with when molten metals are in motion in relation to the temperature. At a high temperature all the metals behave like ordinary (Newtonian) liquids. With lowering of the temperature in some metals (analogous cases—salol and naphtalene), a hard crystalline lattice makes its appearance through which the portion of the liquid which has not had time to crystallize percolates; in others (an analogous case—paraffin) crystallization is followed by the appearance of a large number of crystalline grains, reducing the melt to a pudding-like consistency. The first case can be looked upon as the motion of a liquid through a porous filter with the porosity changing with time, and the second as the motion of a Newtonian liquid of the type of colloid solutions. Formulas are also given for the coefficients of resistance for the motion of a liquid in tubes with smooth and rough surfaces.

I. L. Povkh

Courtesy Referativnyi Zhurnal, USSR

4822. Rossert, R., Critical depth in a channel of trapezoidal cross section (in German), *Bauingenieur* **34**, 2, p. 59, Feb. 1959.

Author has extended the conventional derivation for critical depth (maximum discharge for given energy or minimum energy for given discharge) in a rectangular channel to a trapezoidal cross section with unequal side slopes.

Corresponding to rectangular channel equations

$$y_c = (2/3)\,H \quad [1] \qquad \text{and} \qquad y_c = [\alpha(Q^2\max/b^2g)]^{1/3} \quad [2]$$

author's derivation yields

$$y_c = 0.4H + 0.2\sqrt{[3s/(m+n) - 2H]^2 + 20sH/(m+n)} - 0.6s/(m+n) \quad [3]$$

and

$$y_c = 1/[s + y_c(m+n)/2]\,[\alpha\,(Q^2/g)[s + y_c(m+n)]^{1/3} \quad [4]$$

from which one may readily solve for the maximum discharge,

$$Q_{max} = g\sqrt{[sy_c + [(m+n)/2]y_c^2]^3/[s + (m+n)\,y_c]} \quad [5]$$

No experimental verification is given or indicated.

In the above equations,

y_c = critical depth (Reviewer's note: Author used t_{gr}, the German convention)

H = specific energy

Q = discharge

b = width of rectangular channel

s = bottom width of trapezoidal channel

m, n = side slopes of trapezoidal channel (1 : m = vertical : horizontal)

g = acceleration of gravity

α = factor allowing for nonuniform distribution of velocity in the section

This information may be available somewhere in American engineering literature, but reviewer is unaware of it for *unequal* side slopes.

A. C. Ingersoll, USA

4823. Korol'kov, A. M., The flow of metals and alloys in channels (in Russian), Gidrodinamika Rasplavl. Metallov, Moscow, Akad. Nauk SSSR, 1958, 150–157; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11822.

Data are furnished for the fluid flow of a number of metals and alloys and an analysis is given of the influence of different physicochemical characteristics on the magnitude of the fluid flow. The author concludes, after analyzing the experiments made with zinc, aluminum, tin and lead, that the principal factor determining fluid flow is the heat of crystallization. The curves for the fluid flow of alloys (lead-tin, tin-zinc and aluminum-magnesium) in comparison with the structural diagrams show that the maximum values for fluid flow are to be found in pure metals, and the minimum in alloys in which the heat of crystallization is not liberated at the same time. It is also shown that viscosity is not a determining influence in fluid flow. In conclusion a deduction is put forward that fluid flow of metals and alloys depends first of all on the thermophysical properties and on the situation of the alloys on the structural diagram.

I. L. Povkh

Courtesy Referativnyi Zhurnal, USSR

4824. Morris, A. W., Radial distributions of temporal-mean peripheral velocity and pressure for fully developed turbulent flow in curved channels, *ASME Trans.* **82** D (*J. Basic Engng.*), 3, 528–538, Sept. 1960.

Experimental data as indicated in title are presented for turbulent flow of water in two closed curved channels of rectangular cross section, large depth-to-width ratio and 360° change in direction. For one of the channels, above a certain Reynolds number, the presence of an "anomaly" of the flow consisting of a "belt" of faster moving fluid is reported, and a means of suppressing it is indicated. Reviewer believes that, from the point of view of basic research, a thorough study of such an "anomaly" may prove to be more interesting than its suppression.

E. O. Macagno, USA

4825. Kobori, T., Analysis of waterhammer in case the water level of reservoir fluctuates, *Bull. JSME* **3**, 12, 431–437, Nov. 1960.

Author presents analysis of waterhammer pressures, taking account of effect of changes in water level in surge tanks on rapid valve closure or power failure. Good agreement with conventional method of calculation is found for high-head plants. The author notes that important differences may occur in low-head installations. No examples or data are presented to substantiate author's contention.

D. W. Appel, USA

4826. Stepanoff, A. J., Cavitation in centrifugal pumps with liquids other than water, *ASME Trans.* **83** A (*J. Engng. Power*), 1, 79–90, Jan. 1961.

This latest discourse of the thermodynamic aspects of cavitation with a correlating hydrodynamic commentary, started in this country by the author and Mr. Stahl [*ASME Trans.* **78**, 8, 1691–93, Nov. 1956] continued by Salemann [*ASME Trans.* **81** D (*J. Basic*

Engng. **2**, 167–180, June 1959; AMR **13** (1960), Rev. 1902] is a practicable summary of the art for pump designers and users. Detailed test data, with one 1.5-in, two 2-in. and one 3-in. pump, in a specific speed (N_s) range of 640 to 1620 with suction specific speeds (S) of 6800 to 11500, are given for water from 70°, 400°F, butane and Freon 11. Based on these, the thermal cavitation parameter B is shown on a chart as a function of vapor pressure, the test points with different symbols for the various fluids indicated on the marked S lines. A modification of B called B_m and some simplification led to a final NPSH correction chart with vapor pressure as abscissa, distinct lines for the different liquids and correction diagram for S. The article "should be considered as a first step toward learning about the behavior of the different liquids under cavitation conditions."

A. Hollander, USA

4827. Szalay, M., Flow-velocity measurement in the laboratory, using spheres (in Hungarian), *Hidrológiai Közlöny* **40**, 3, 204–207, June 1960.

Author discusses the hydrodynamic resistance of spheres, and describes two instruments based on it which are intended for laboratory measurements in experimental channels having glass walls through which the instruments can be observed. One instrument is intended for the simultaneous measurement of flow velocity in a channel at several levels, from its bottom to its upper surface; it consists of a number of buoyant spheres (in the actual instrument 0.5-cm diameter) threaded on a thin thread at equal distances from one another, like in a rosary; the end of the thread is fixed at the bottom of the channel, and the spheres take on positions in a slanting line, the top sphere being also submerged. After equilibrium is reached, the string of spheres can be photographed, and show a broken line; at each sphere the four forces—buoyancy, hydrodynamic drag, and the tensional forces on the adjoining two threads—are in equilibrium; the buoyance and the drag coefficients are known, the angles of the threads can be measured from the photograph, and from these data and a chart showing the relation of drag force to flow velocity, the flow velocity for each sphere can be determined.

In the second instrument only one sphere is used, attached to a string, the other end of which is fixed to the origin of a protractor quadrant. When submerged in a flowing stream, the three forces—buoyancy, drag force on sphere, and the force on the string—are in equilibrium; from the slope of the string, with the aid of tabulated data, the flow velocity can be determined; or, the quadrant can be calibrated directly in velocity units. The operational requirements, advantages, disadvantages, and error sources of the two instruments are discussed.

K. J. DeJuhasz, USA

Incompressible Flow

(See also Revs. 4604, 4822, 4824, 4848, 4890, 4893, 4898, 4907, 4911, 4912, 4913, 4936, 4989, 4992, 5055, 5109, 5151, 5167)

4828. Venkates, H. G., Motion of a viscous liquid past an ellipsoid, *Physics of Fluids* **4**, 1, 33–39, Jan. 1961.

Exact integration of the Navier-Stokes equations is performed by postulating body forces which cancel a combination of vorticity and viscous stress terms. This enables the pressure to be found from an equation analogous to Bernouilli's, and the velocity from continuity and the postulated body forces. For physically important flows the body force must be made to vanish in the limit; thus the results are exactly valid only for very special flows. (Irrotational flow would be one of these.)

Particular results are computed for potential flow and slow viscous flow past an ellipsoid. The boundary layer is also found as a limiting case.

W. P. Graebel, USA

4829. Finn, R., An energy theorem for viscous fluid motions (in English), *Arch. Rational Mech. Anal.* **6**, 5, 371–381, Dec. 1960.

The problem considered is a body in uniform motion through an infinite incompressible medium which has reached steady state (being time-independent). Seven theorems are proved in connection with the total kinetic energy, the behavior of the velocity of medium and the force exerted on a smooth closed surface.

W.-H. Chu, USA

4830. Lighthill, M. J., Note on the swimming of slender fish, *J. Fluid Mech.* **9**, 2, 305–317, Oct. 1960.

In this paper inviscid slender-body theory is applied to deformable bodies. From the resulting expressions for the mean thrust and rate of shedding of kinetic energy of lateral fluid motion the efficiency of swimming motions can be deduced if an appropriate cycle of body movements is prescribed. As the functions describing these movements must contain "recoil" terms so that the equations of motion may be satisfied, a general solution of the problem requires extensive numerical calculations. Conditions for efficient swimming are also discussed. For the purposes of the discussion, recoil is not taken into account by the author. He finds that the thrust depends only on conditions at the end of the tail and that the lateral velocity of the fluid at the tail must be small compared to the rate of lateral displacement of the body, i.e., on the average these must have opposite signs. One type of motion which satisfies this condition is a wave traveling along the body at a speed of around 5/4 of the swimming speed, with the amplitude increasing toward the tail and of wavelengths of less than 1/2 of the body length. The theory utilizes the concept of "virtual mass" of cylindrical sections of the body and consequently absence of boundary-layer separations is implied.

A. Fejer, USA

4831. Maroti, L. A., Deak, G., and Kreith, F., Flow phenomena of partially enclosed rotating disks, *ASME Trans.* **82**D (*J. Basic Engng.*), 3, 539–552, Sept. 1960.

Interesting exploratory investigation of the unsteady flow induced by a disk rotating between two parallel stationary walls with diameter larger than disk diameter. Periodic flow reversals were detected contrary to expectation and were observed using different techniques for Reynolds numbers $\omega R^2/\nu$ from 3×10^5 to 6×10^6 and for clearance-to-diameter ratios from 0.0125 to 0.0625. The flow was found steady when diameter of two walls was equal or less than disk diameter. Reviewer joins authors and some of the discussers of this paper in that additional research is needed for a better understanding of instability phenomena such as those reported.

E. O. Macagno, USA

4832. Poll, W. H., and Payne, L. E., On Stokes flow about a torus, *Mathematika* **7**, 13, 78–92, June 1960.

Application of earlier work by the authors on Stokes flow past axisymmetric bodies [*J. Fluid Mech.* **7**, p. 529, 1960; see also AMR **13** (1960), Rev. 2913] with an additional complication owing to the fact that the line of symmetry does not meet the body. This difficulty is resolved by noting that the pressure is single-valued. Solution is given in toroidal coordinates as a series of Legendre functions $P_{n-\frac{1}{2}}$ and their derivatives. Expressions are given for the drag of the body and the pressure and the stress components at any point.

Reviewer would have liked to have seen a numerical example. It would have been interesting, for instance, to compare the results (and the convergence of the series) with those obtained for inviscid flow by Sternberg and Sadowsky [AMR **7** (1954), Rev. 536].

J. C. Cooke, England

4833. Pukhnachev, V. V., Group properties of the Navier-Stokes equation in the plane case (in Russian), *Z. Prikl. Mekh. Tech. Fiz.* **1**, 83–90, May/June 1960.

Author sets out from the two-dimensional Navier-Stokes equations for an incompressible fluid without body-force, and constructs eight linearly independent basic groups of differential operators which lead to Lie transformations and afford the means of constructing invariant solutions of the Navier-Stokes equations. Author then enumerates and considers in detail the 26 two-parameter subgroups and the invariant solutions to which they lead. Many of these are new and all are systematically obtained.

L. M. Milne-Thomson, USA

4834. Pearson, J. R. A., The instability of uniform viscous flow under rollers and spreaders, *J. Fluid Mech.* **7**, 4, 481–500, Apr. 1960.

The waved or ribbed surface formed when a thin film of viscous fluid passes through a small gap is governed by viscous, inertia, gravity and surface tension forces.

Their relationship can be expressed in nondimensional terms and the problem is analyzed in terms of lubrication theory. Predictions are compared with experimental observations for the particular case of a spreader in the form of a wide-angled wedge. Agreement is obtained over a wide range of a parameter relating surface tension with viscosity and velocity. For low values of this parameter, other conditions involving only viscous and pressure forces determine the behavior observed. An incidental conclusion from this work is that what are commonly regarded as brush marks are not specifically related to the bristles but are a further manifestation of behavior common to most spreading devices.

F. T. Barwell, England

4835. Butler, L. H., The hydrodynamic effect between approaching surfaces with interposed viscous fluid films, and its influence on surface deformations, *J. Inst. Petroleum* **46**, 435, 63–73, Mar. 1960.

Surface effects of approaching metallic (originally) flat specimen with fluid films sandwiched between are investigated. Significant effects of grain distortion as well as dishing of the surfaces as a whole are demonstrated.

This research yields some interesting results, but of course the field of real interest still lies with the dynamic aspects of this same problem, with actual measurements of the pressure and distortions being made. E. K. Gatcombe, USA

4836. Luneau, J. L., Influence of acceleration upon the resistance of movement in fluids (in French), Publ. Scient. Tech. Min. Air, France no. 363, 152 pp., 1960.

An experimental study of the drag of uniformly accelerated bodies in incompressible flow. The results for bodies of simple geometric shape are explained partly on the basis of the virtual mass concept and partly by changes in flow regimes. It is found that transition from laminar to turbulent flow in the boundary layer is delayed by a positive acceleration.

Tests are also made on an airfoil section in uniformly accelerated motion. At low incidences the drag coefficient is slightly increased and the lift coefficient slightly reduced. However, stalling is delayed and, at high incidences, the lift coefficient is greatly increased while the drag coefficient is slightly reduced. Similar increases in lift coefficient are found to occur in uniformly decelerated motion.

G. A. Bird, Australia

4837. Singh, D., Superposable flows of the type curl $q_1 = \lambda_2 q_2$ and curl $q_2 = \lambda_1 q_1$, *Ganita* **10**, 2, 83–93, Dec. 1959.

If scalars λ_1 and λ_2 are independent of the space variables, they are also time-independent fixed constants, a known result. Here, the condition that they be functions of one rectangular coordinate and time is obtained. C. Ablow, USA

4838. Rott, N., On the viscous core of a line vortex, Parts 1 and 2, *ZAMP* **9b**, 5/6, 543–553, Mar. 1958; **10**, 1, 73–81, Jan. 1959.

In Part 1 the author considers the problem of a potential flow directed away from an axisymmetric stagnation point and with a superimposed vortex centered about the Z-axis. The case of a potential vortex (constant circulation) is first discussed and shown to be an unsatisfactory solution since the circumferential velocity becomes infinite at the origin. A new steady-state solution is then obtained which eliminates this difficulty. This solution consists physically of a viscous core which is aligned with the axis of the stagnation point flow. Outside this core the flow reduces to that produced by the previously mentioned potential vortex. A very interesting discussion is presented with respect to physical phenomena such as a bathtub vortex and tornadoes. A further solution is obtained for unsteady circulation.

Part 2 deals with evaluating the temperature distribution for the steady-state problem considered in part 1. The assumption is made that "high Reynolds number" circulatory motion exists, and this permits the assumption that viscous dissipation due to the basic stagnation point flow is negligible compared to that produced by the circulatory motion. R. Cess, USA

4839. Schroder, H. J., Viscous flow through diffusing scrolls of arbitrary cross-sectional shape (in German), *Forsch. Geb. Ing.-Wes.* **25**, 6, 182–189, 1959.

4840. Legchenko, I. G., The problem of the flow around an elliptical cylinder by a current of incompressible liquid with a pair of attached vortices (in Russian), *Izv. Sibirsk. Otd. Akad. Nauk SSSR* no. 9, 52–59, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11730.

A solution is furnished for the known problem on the flowing about by a plane flow of ideal liquid of an elliptical cylinder with two attached vortices. It is shown that when utilizing the conditions for the break in the currents, given by A. A. Kosmodem'yanski [*Tr. TsAGI* (1957), no. 317] calculations were made for the coordinates of the points of break of the currents and the angles of break of the currents for ellipses with half-axis relations of $\lambda = 2/3$ and $\lambda = 1/2$ and that these magnitudes proved to be close to those which the author obtained experimentally.

E. L. Blokh

Courtesy Referativnyi Zhurnal, USSR

4841. Tsakonas, S., and Jacobs, W. R., Potential and viscous parts of the thrust deduction and wake fraction for an ellipsoid of revolution, *J. Ship Res.* **4**, 2, 1–16, Nov. 1960.

Deeply submerged hull-propeller interaction was studied by replacing the axisymmetric hull by an axial source-sink distribution (Havelock), and the propeller by a sink disk (Burgers). The integrated Lagally force on the entire axial source-sink system then represents the interaction thrust-reduction, or drag-augmentation, of the potential field. Effect of viscosity, i.e., boundary layer, is determined by using Lighthill's method, e.g., distributing equivalent surface sources having the strength proportional to the displacement thickness and its derivative, and an equivalent sink downstream of hull, but upstream of propeller to satisfy closure condition. The wake is now replaced by a solid core of radius equal to displacement thickness at the stern, outside of which the field is again potential. This radius is less than the radius of propeller hub, thus propeller is operating in a potential field.

The parametric dependence of this hull-propeller interaction on the body-propeller clearance, body slenderness, afterbody geometry, and Reynolds number are explicitly exhibited. For inviscid case, the thrust reduction was indeed obtained in closed-

form expression. For viscous case, this is found to be insensitive to the detailed information of boundary-layer growth, i.e., several different methods of boundary-layer evaluation (Millikan, Granville, Hickling, Lavrentieff) yield practically identical results. H. S. Tan, USA

4842. Goldshtik, M. A., Secondary flow arising about a sphere rotated in a viscous fluid (in Russian), *Inzbener. Fiz. Zb.* **3**, 3, 79–82, Mar. 1960.

An approximate solution for the equation of motion induced by a sphere rotating in a viscous incompressible fluid is found.

From the expression obtained the streamlines in a meridional cross section can be constructed.

From author's summary by A. N. Petroff, USA

4843. Kapur, J. N., On axially symmetric superposable flows: Part 2, *Bull. Calcutta Math. Soc.* **52**, 1, 14–24, Mar. 1960.

Author pursues his study on the properties of axially symmetric viscous flows (see AMR **13** (1960), Rev. 4662) and discusses the following new cases: (a) steady axially symmetric Beltrami viscous flows; (b) steady axially symmetric self-superposable viscous flows; (c) most general poloidal and toroidal velocity fields superposable on each other; (d) most general velocity field superposable on a toroidal field; (e) possibility of axially symmetric superposable motions of the type: curl $\vec{q}_1 = \lambda_2 \vec{q}_2$, curl $\vec{q}_2 = \lambda_1 \vec{q}_1$ (with constant values for λ_1 and λ_2) in inviscid fluids. For viscous fluids such flows do not exist. G. Sestini, Italy

4844. Ringleb, F. O., Two-dimensional flow with standing vortexes in ducts and diffusers, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 921–928, Dec. 1960.

The conditions for the equilibrium of two vortexes in a two-dimensional flow through a duct or diffuser are derived. Potential flow considerations and a few basic results from viscous-flow theory are used for the discussion of the role of cusps as separation, control and trapping devices for standing vortexes. The investigations are applied to cusp diffusers, especially with regard to the wind tunnel of Princeton University.

D. Gh. Ionescu, Roumania

4845. Ovchinnikov, O. N., Axisymmetric streamlined bodies of revolution in vortex flow (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekb. Nauk, Mekb. i Masbinostroenie* no. 6, 19–23, Nov./Dec. 1960.

Author considers axisymmetrical steady flow for which the energy per unit mass is proportional to the stream function so that at distance y from the axis the velocity is of the form $U + ky^2$ parallel to the axis. The behavior of a half-body of revolution, a spheroid, and a sphere disturbing the above flow are studied and diagrams show the streamlines and velocity-distribution.

L. M. Milne-Thomson, USA

4846. Levy, S., Steam slip—Theoretical prediction from momentum model, *ASME Trans.* **82** C (*J. Heat Transfer*), 2, 113–124, May 1960.

Theoretical equations governing slip effects in forced circulation of boiling water are derived. A simplified momentum model is postulated based on equal friction and head losses of two phases, yielding a relation between quality and voids. The equations indicate that steam slip is dependent upon channel geometry, inlet water velocity, and rate of heat addition. Good agreement is obtained with available experimental results in horizontal and vertical test sections with and without heat addition at pressures from 12 to 2000 psia. Discussion of the model in terms of nonquasi-steady-state unbalances of friction and head losses of the two phases explains experimental deviations from the predictions and the effects of water inlet velocity, channel geometry and rate of heat addition. The simplified model is applied to calculating two-phase pressure drops. C. F. Bonilla, USA

4847. Robinson, A. R., On two-dimensional inertial flow in a rotating stratified fluid, *J. Fluid Mech.* **9**, 3, 321–332, Nov. 1960.

An inviscid, incompressible, stratified fluid is bounded above by a horizontal plane and below by finite-amplitude parallel ridges, the ridges being in rotation about a vertical axis. The nonlinear equations of motion are exhibited, governing parameters identified, and solutions obtained in cases where one or the other parameter is small. Results are applicable to bulk motions of the deep ocean. Solutions illustrate many analytic methods for handling nonlinear equations. C. M. Ablow, USA

4848. Robertson, J. M., and Fraser, H. R., Separation prediction for conical diffusers, *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 201–209, Mar. 1960.

Empirical method of turbulent boundary-layer analysis due to D. Ross [AMR **7** (1954), Rev. 3973] is used to determine separation for conical diffusers with partially developed entry flow. Major parameters are shown to be the initial momentum thickness Reynolds number $R\theta_0$, and a distance parameter $L^3 = x^3/\theta_0 R_0$, where x is axial distance from origin, R_0 the initial diameter, and θ_0 the initial momentum thickness. Tests for $0.36 \times 10^4 \leqslant R_{\theta_0} \leqslant 1.7 \times 10^4$ show that separation prediction is slightly conservative. Analysis used limits method to $x/\delta_0 < 60$ (where δ_0 is initial displacement thickness) $\theta_0/R_0 < 0.06$ at entry and < 0.1 at exit, and to flows in which adverse pressure gradient predominates over skin-friction effects. Method also is not applicable to large-angle diffusers. Approximate design method is given in appendix and paper includes discussion by Hydraulic Conference members. Method is considered to give a useful engineering solution for small-angle diffuser design. R. N. Cox, England

4849. Singh, D., On superposable motions on a motion outside a rectilinear vortex filament, *Ganita* **10**, 2, 49–56, Dec. 1959.

A formula is derived for all rotational two-dimensional flows superposable on irrotational vortex flow. A special case gives the initial viscous flow outside a rotating collapsing cylinder.

C. M. Ablow, USA

4850. Iordanskii, S. V., Equations of motion of a fluid containing gas bubbles (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **3**, 102–110, 1960.

Author derives the equations of mean motion of a liquid with a small concentration of tiny gas bubbles; he particularly considers the internal motion of the liquid due to the motion and the pulsations of the bubbles, under an acoustic approximation for the equation of state of the liquid. L. Trilling, USA

4851. Razi, A., The velocity of the circulation and distribution of air in large chambers (in Russian), IX Mezhdunar. Kongress Kholoda, 1955; Moscow, Gostorgizdat, 1958, 171–175; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11628.

Results are given of a large number of experiments whose object was to obtain the distribution of the velocity of the air entering the chamber through an orifice along the axis of the air current and in a perpendicular direction. The experiments were carried out both in isothermal and nonisothermal conditions. It was shown that of all the known formulas for the residual velocity along the jet v the most accurate appeared to be the Tuive and Priester empirical formula

$$v = K \frac{G}{x\sqrt{A}}$$

where G is the jet's discharge, x the distance along the axis of the jet in meters, A the actual section of the orifice, K a coefficient.

The ratio between the maximum and the mean velocity along the radius of the section of the jet is 2.5. The angle of the jet may have values of from 14° to 26° for initial velocities of from 5 to 20 m/sec. The mean value for the angle of the jet for the whole route covered was 10°. L. Ya. Khaskin

Courtesy Referativnyi Zhurnal, USSR

4852. Yih, C.-S., Unsteady transfer of momentum and heat between concentric cylinders, *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 210–216, Mar. 1960.

Paper analyzes the unsteady motion of a viscous fluid between two concentric cylinders. The viscosity is assumed to be constant, a fact which permits the velocity distribution to be studied independently of the temperature distribution.

For the first problem the motion equations are written in polar coordinates, and finally the differential equation which gives the velocity distribution function is obtained. By linearizing this equation, the solution is resolved into two parts; a part which represents the solution corresponding to the steady regime (for the time $t \longrightarrow \infty$), and a solution of the complete equation which is given under the form of a development into series of Bessel functions (by applying the separation of variables). In this way several problems are solved, e.g., the motion due to a sudden or gradual revolution of one of the cylinders, the motion due to an axial pressure gradient, and so on. By using a similar method the energy equation is also solved, the increase of temperature due to steady or unsteady friction stresses being determined for simplified boundary conditions.

Since, as the author himself notes, the results are to be applied to problems of hydrodynamic lubrication, the particularization of the theory for the case in which the lubricant film thickness has low values with respect to the radii of cylinders would have been interesting. Thus a brief examination of the results shows that the velocity distribution tends rapidly towards $t \longrightarrow \infty$ or follows the motion of cylinders while the temperature distribution remains variable in time. An examination of the problem in which the viscosity varies with the temperature (as in lubricating oils) would have been also of interest.

V. N. Constantinescu, Roumania

Compressible Flow (Continuum and Noncontinuum Flow)

(See also Revs. 4895, 4897, 4907, 4918, 4926, 4948, 4949, 4950, 4956, 4957, 5003, 5034, 5062, 5073, 5094)

4853. Clarke, J. F., The linearized flow of a dissociating gas, *J. Fluid Mech.* **7**, 4, 577–595, Apr. 1960.

Equations describing steady two-dimensional flow of an ideal dissociating diatomic gas are linearized for the case in which the free-stream gas is in chemical equilibrium, deviations from free-stream conditions are small, and chemical reaction rates are finite. The resulting equations are applied to the case of supersonic flow around a sharp corner with small turning angle. Pressures, densities, temperatures, and compositions at the wall are calculated as functions of distance from the corner. Comparisons with results of exact calculations indicate that the linearized equations predict to good approximation absolute values of temperatures and concentrations as well as values of pressures and densities relative to pressure and density immediately downstream of the corner. The linearized theory overestimates, however, the reductions of pressure and density at the corner. Reviewer would encourage attempts to obtain experimental data for comparison with these calculation results.

E. L. Knuth, USA

4854. Moisil, G., Observations on the linearized equations of viscous compressible fluids (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **12**, 1, 41–47, 1961.

Author's method of associated matrices [see Gr. C. Moisil, *Acad. Repub. Pop. Romane, Bul. Stiint., Sect. Stiinte Mat. si Fiz.* **4**, 319–401, 1952] is applied to the system

$$\frac{\partial^2 u}{\partial t^2} - a^2 \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial Z}\right) = \frac{\nu}{3}\frac{\partial^2}{\partial x \partial t}\left(\frac{\partial u}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial w}{\partial Z}\right) + \nu\Delta\frac{\partial u}{\partial t} \qquad [1]$$

It is shown that the velocity components u, v, w satisfy the equation

$$\left[\left(a^2 + \frac{4\nu}{3}\frac{\partial}{\partial t}\right)\Delta - \frac{\partial^2}{\partial t^2}\right]\left[\nu\Delta - \frac{\partial}{\partial t}\right]\frac{\partial \Phi}{\partial t} = 0, \qquad [2]$$

the vortex components ξ, η, ζ satisfy Eq. $(\nu\Delta - \partial/\partial t)\partial\Psi/\partial t = 0$, and the volumetric dilation θ satisfies Eq. $[(a^2 + \frac{4\nu}{3}\partial/\partial t)\Delta - \partial^2/\partial t^2]\,\theta = 0$. The velocity components can be represented by means of a Galerkin vector, whose components are integrals of Eq. [2]. It results that each movement described by system [1] can be resolved into three movements: a stationary movement, an incompressible movement and an irrotational one.

D. Ionescu, Roumania

4855. Rosen, G., Exact solutions for the one-dimensional viscous flow of a perfect gas, *Physics of Fluids* **3**, 2, 191–196, Mar./Apr. 1960.

Following the method previously set forth by the author [AMR **13** (1960), Rev. 5217], the conservation equations governing unsteady one-dimensional viscous flow of a perfect gas are expressed in a form independent of velocity. It is then shown that these equations admit three symmetry properties: a pressure-time symmetry, a stress symmetry, and a displacement symmetry. Making use of each of these symmetry properties, fundamental exact solutions of the governing partial differential equations are obtained and discussed. R. D. Cess, USA

4856. Hayes, W. C., Jr., and Henderson, W. P., Some effects of nose bluntness and fineness ratio on the static longitudinal aerodynamic characteristics of bodies of revolution at subsonic speeds, NASA TN D-650, 65 pp., Feb. 1961.

The effects of a systematic variation of nose shape and fineness ratio on the longitudinal aerodynamic characteristics of bodies of nose shapes were investigated, representing five corner radii which varied from 0 to 50 percent of the body diameter and three face radii which varied from 50 percent of the model diameter to infinity. The complete models had fineness ratios of 0.50 to 2.00. In addition, the effects of boattailing the afterbody and removing or varying the transition strips which had been attached to initiate a turbulent boundary layer were noted. Results are presented for an angle-of-attack range from −4° to 24° for Mach numbers from 0.25 to 0.80, and indicate that small variations of the model nose can produce large variations in the static longitudinal aerodynamic characteristics of the body. These variations may in turn be moderated by an increase in the model fineness ratio.

From authors' summary

4857. Laval, P., Newtonian flow on conical surfaces with nonzero angle of attack (in French), ONERA Pub. 73, 5–16, Nov./Dec. 1959.

4858. Kapur, J. N., Transverse component of velocity in a plane symmetrical forced jet of a compressible fluid, *Bull. Calcutta Math. Soc.* **51**, 1, 34–38, Mar. 1959.

4859. Ryzhov, O. S., On the transition from subsonic to supersonic velocities in Laval nozzles, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **24**, 2, 534–540, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

This investigation deals with three-dimensional mixed flows of an ideal gas in Laval nozzles. The study is concerned particularly with the form of the surface of transition in the case when the velocity at the center of the flow approaches the velocity of sound, while the derivative of the velocity in the direction of the canal axis at that point vanishes. A theorem is derived which is a generalization for three-dimensional motion of a well-known theorem of Frankl and Gortler, valid in the cases of plane-parallel and axisymmetrical gas streams. On the basis of this theorem two possible types of flows in the neighborhood of the throat of a nozzle, and the possibility of transition of one type into the other, are discussed.

From author's summary by A. J. Chapman, USA

4860. Iur'ev, I. M., On the theory of plane gas flows, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 1, 287–299, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Starting from a certain simple tractable solution of a system of equations of the Chaplygin type for the plane motion of a gas, a method is presented by which other systems of equations of a type containing arbitrary constants in their coefficients can be obtained. By selecting the constants it is possible to obtain good approximations to the equations of adiabatic gas flow over a wide range of velocity variation. The technique generalizes the work of Peres so that such important properties of the initial flow as continuity of the subsonic flow into the supersonic domain and uniformity of the flow at infinity are preserved. The method is applied to the calculation of the transonic part of the flow through a family of nozzles.

From author's summary by M. Epstein, USA

4861. Monta, W. J., Howard, P. W., and Czarnecki, K. R., Effect of nose bluntness on transition for a cone and a hollow cylinder at Mach numbers 1.41 and 2.01, NASA TN D-717, 34 pp., Apr. 1961.

The results of the investigation indicate that for relatively small ratios of bluntness height to transition distance on the sharp-nosed cone or cylinder there was little, if any, effect of blunting on transition. For somewhat larger values of this bluntness ratio there was a favorable effect of blunting on transition on the cylinder but little effect on the cone. For relatively large values of the ratio the effects of blunting on both the cone and cylinder were adverse. Increasing the Mach number had a mitigating effect on this adverse effect on blunting.

From authors' summary

4862. Bulygina, E. V., Approximate solution to the problem of supersonic flow past a triangular wing (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **1**, 142–144, May/June 1960.

Source-potential theory of linearized aerodynamics is used to calculate flow over a delta wing with completely subsonic leading edge in a supersonic stream.

The potential function is expressed as an integral over a region consisting partly of a portion of wing surface, where source strength is known, and partly of a region between the leading edge and the Mach cone from the vertex. The integral over the latter region, where source strength is unknown, can be expressed in terms of other integrals involving regions similar in nature, but smaller in size. A recurrence relation is thus set up leading to an infinite series of known integrals, whose sum represents the required unknown integral.

Convergence of the series is proved and even low-order approximations to its sum are shown to be in good agreement with results of exact conical flow theory.

A. H. Armstrong, England

4863. Korst, H. H., Chow, W. L., and Zumwalt, G. W., Research on transonic and supersonic flow of a real fluid at abrupt increases in cross section (with special consideration of base drag problems), AFOSR TR 60-74, (Univ. Illinois, Engng. Experiment Station), Mech. Engng. Dept. ME TR-392-5), 151 pp., Dec. 1959.

A general theory for pressures and temperatures in separated transonic and supersonic flows is developed using a physically perceptive model composed of simplified flow components. The principle of interaction between dissipative and free-stream flow regions allows one to distinguish between inviscid flow configurations and superimposed jet mixing regions. Treatment of the latter is based on concepts developed in ME-TN-392-1, and on tabulated results (including theoretical Stanton numbers and drag coefficients) for fully developed isoenergetic and nonisoenergetic, constant pressure turbulent ($Pr_t = 1$) compressible two-dimensional mixing regions.

A great variety of problems in internal and external flow is analyzed theoretically, ranging from the classical base pressure problem to the performance of supersonic ejectors and the detailed study of the jet-slipstream interaction (afterbody-) problem in jet engine and rocket propulsion.

From authors' summary by G. A. Altmann, Germany

4864. Yur'ev, I. M., Approximate solution of the basic boundary-value problem of plane supersonic flow of a gas (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **25**, 188–196, 1959.

Author constructs an approximate solution of the fundamental boundary-value problem of plane supersonic flow. The starting point is the Chaplygin equation for the streamfunction of the flow. The method used is that of Laplace. Introducing a new function ψ_1 into the Chaplygin equation, author proposes such a separation of variables that the remodeled Chaplygin equation splits into two ordinary equations which are representable by means of exponential functions. The power coefficients of these exponential functions are the integrals of complicated trigonometric functions. Having these expressions, the author returns to the function ψ_1 and next to the original streamfunction. The expression so obtained is very complicated. In a similar way the author derives the expressions for the differentials dx and dy.

As the first application of this method the author attacks the Cauchy problem. The entire formalism is presented in the most general form, using symbolic notation. At the end of his paper the author discusses the paper by G. A. Dombrowski [*Doklady Akad. Nauk SSSR* **107**, no. 6, 1956], showing that the results obtained by Dombrowski are identical with the results obtained by himself in 1951.

M.Z.v. Krzywoblocki, USA

4865. Pokhozhaev, S. I., A question relating to supersonic flow (in Russian), *Trudi Mosk. Fiz.-Tekhn. In-ta* no. 1, 167–172, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11450.

A supersonic plane jet of ideal gas flows out of a nozzle with parallel walls into an immovable medium at a reduced pressure p_3. The solution resolves into a series by steps of the small parameter $\epsilon = (p_1 - p_3)/p_1$ where p_1 is the pressure in the outlet section of the nozzle; in the calculations terms are retained which contain ϵ up to a degree not higher than the second. The rarefied waves leaving the edges of the nozzle are investigated and with the aid of the equation for the free surface of the jet, evolved by Lin [C. C. Lin, *J. Math. Phys.* **33**, 2, 117–134, 1954; AMR **8**(1955), Rev. 717], proofs are furnished that after reflection the converging waves will not be symmetrical with the diverging waves. Here a

deduction is made that when the values for ϵ are sufficiently small shock waves make their appearance in the jet. A similar deduction was made, using another and more exact method, by G. A. Dombrovskii for a case of calculation when $\epsilon = 0$ [*Doklady Akad. Nauk SSSR* **113**, 1, 58–61, 1957]. M. I. Gurevich

Courtesy Referativnyi Zhurnal, USSR

4866. Krasnov, N. F., The method of characteristics and its application to the calculations for the distribution of pressure close to sharp-ended bodies of rotation moving at supersonic speeds under an angle of attack (in Russian), Nekotorye Vopr. Mekhaniki (MVTU **88**), Moscow, 1958, 55–75; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11479.

A deduction is given of the relations for the characteristics of a three-dimensional potential flow in terms of cylindrical coordinates. A general form of the equation for the characteristics is presented, referring to the case of quasilinear differential equations in particular derivatives of the second order with a number of independent variables equal to or more than three; this is followed by an investigation of its particular form for an unsymmetrical flow near a body of rotation, and also of the common nature of the characteristics. A method of calculation is developed for the distribution of velocities when the body is being flown past under an angle of attack. G. I. Bogomolov

Courtesy Referativnyi Zhurnal, USSR

4867. Marvin, J. G., Surface pressures and heat transfer on unswept blunt plates in helium at high Mach numbers, NASA TN D-688, 29 pp., Mar. 1961.

Data were obtained on a flat plate at zero angle of attack with various amounts of leading-edge bluntness at Mach numbers of 12.5 and 14.7. Pressures on blunt plates were correlated by introducing the leading-edge Reynolds number as a parameter. Measured heat-transfer coefficients are compared with values predicted across the laminar boundary layer. Shock-wave shapes were represented by a form of the blast-wave analogy.

From author's summary

4868. Janos, J. J., Loads induced on a flat-plate wing by an air jet exhausting perpendicularly through the wing and normal to a free-stream flow of Mach number 2.0, NASA TN D-649, 21 pp., Mar. 1961.

Measurements were made of loads induced on a flat-plate wing by an air jet exhausting perpendicularly through the wing and normal to the free-stream flow. The investigation was conducted at a free-stream Mach number of 2.0 and a Reynolds number per foot of 14.4×10^6. An axially symmetric sonic nozzle and two supersonic nozzles were employed for the jets. The supersonic nozzles consisted of an axially symmetric nozzle with exit Mach number of 3.44 and a two-dimensional nozzle with exit Mach number of 1.76. The ratio of nozzle total pressure to free stream static pressure was varied from 20 to 110.

Negative loads were induced on the flat-plate wing by all the jets. As the nozzle pressure ratio was increased the magnitude of interference loads due to jet thrust decreased. The chordwise center-of-pressure location generally moved toward the nozzle center line as the pressure ratio was increased.

From author's summary

4869. Ginoux, J. J., Separated supersonic flows, Proc. Heat Trans. Fluid Mech. Inst., Stanford, Calif., June 15–17, 1960; Stanford Univ., 1960, 179–191.

4870. Lomax, H., Two-dimensional, supersonic, linearized flow with heat addition, NASA Memo 1-10-59A, 35 pp., Feb. 1959.

4871. Swigart, R. J., Third-order blast wave theory and its application to hypersonic flow past blunt-nosed cylinders, *J. Fluid Mech.* **9**, 4, 613–620, Dec. 1960.

The inviscid flow behind a cylindrical blast wave and its analogy with hypersonic flow past blunt-nosed cylinders is considered. Sakurai (1953, 1954) obtained a solution for the flow field behind a propagating blast wave by expanding the flow variables in power series of $1/M^2$, where M is the blast-wave Mach number, and determining the coefficients of the first two terms in the series. Here the work is extended to include third-order terms. Third-order theory is shown to improve the prediction of shock-wave shapes and surface pressure distributions on hemisphere-cylinder configurations at $M_\infty = 7.7$ and 17.18.

From author's summary by R. A. Gross, USA

4872. Eula, A., Features and methods of theoretical research for hypersonic flows (in Italian), *Aerotecnica* **40**, 2, 79–93, Apr. 1960.

Characteristics of hypersonic flow are reported. Theory of small perturbations and fundamentals of Newtonian and constant density theories are also presented. Then author presents the general theory of flow in the shock layer and several methods for solution of direct and inverse problems. An outline is given with respect to the corrections to be introduced in the methods in order to take into consideration the real nature of the gas. After an outline of the influence of dissociation on heat transmission, various methods are given for the determination of the hypersonic boundary layer, whether laminar or turbulent, as well as for the calculation of the heat flow. Finally, interference phenomena between the boundary layer and the exterior flow are described, as regards the large increase in boundary-layer thickness and the exterior flow vorticity. From author's summary by A. Balloffet, USA

4873. Muggia, A., On inviscid hypersonic flow near the stagnation point of a blunt body (in Italian), *Aerotecnica* **40**, 2, 112–118, Apr. 1960.

Axisymmetric body with rounded nose is placed in steady axial flow of inviscid compressible fluid, with infinite undisturbed Mach number. Method is based upon a double series development of which several terms are computed and the procedure for determination of each successive term is given. Relationship between shapes of body and shock wave is presented. Distribution of velocity, pressure and vorticity on the surface of body and at stagnation point vicinity are studied. Results are compared with Lighthill theory, as well as with Newtonian theory and its revisions and modifications. Comparisons are also made with results obtained by Mangler. Some investigations are made with respect to convergence of series obtained in study.

From author's summary by A. Balloffet, USA

4874. Mirels, H., Approximate analytical solutions for hypersonic flow over slender power law bodies, NASA TR R-15, 24 pp., 1959.

Zeroth-order and first-order solutions are found for the two-dimensional and axisymmetric hypersonic flow about certain power law bodies within the limitations of inviscid, slender-body theory. The bodies range from the constant-energy case of a blunt-nosed plate or right, circular cylinder to the well-known wedge or cone case. Approximate analytical results are obtained from the assumption of the form of the zeroth-order normal or radial velocity component. This is based on the asymptotic expression derived for the corresponding stream function near the body. Three constants which are introduced are determined by appropriate satisfaction of the shock boundary conditions. Consequently, the energy and continuity equations are identically satisfied, but the momentum is not except at the shock.

The first-order solutions follow a somewhat similar procedure and together with the zeroth-order solution furnish relatively simple formulas for the surface pressure, shock shape and drag. Numerical results and comparisons are also given. In general, the accuracy is best for flows close to the wedge or cone case with a ratio of specific heat near unity. Numerical discrepancies with the first-order results of T. Kubota are noted [AMR 11 (1958), Rev. 208]. Some very interesting characteristics for flow fields associated with other power law shocks are also given.

D. E. Ordway, USA

4875. Cubbison, R. W., Anderson, B. H., and Ward, J. J., Surface pressure distributions with a sonic jet normal to adjacent flat surfaces at Mach 2.92 to 6.4, NASA TN D-580, 29 pp., Feb. 1961.

An investigation was made to determine the interference effects on surface pressure distributions caused by a sonic jet exiting normal to the surface. Two configurations, a flat plate and an arrow-wing reentry-type vehicle, with sonic nozzles near the leading edge were tested over a range of pressure ratios and Reynolds numbers for Mach numbers from 2.92 to 6.4.

The data indicate that jet pressure ratio had considerable effect on the pressure levels and distributions on both configurations. Also, for a constant jet pressure ratio, the free-stream Mach number effect on the distributions and levels was quite large. Over the limited range investigated, the effect of Reynolds number at constant Mach number and pressure ratio was small compared to the Mach number and pressure ratio effect.

From authors' summary

4876. Richtmyer, R. D., Taylor instability in shock acceleration of compressible fluids, *Comm. Pure Appl. Math.* **13**, 2, 297-319, May 1960.

Paper considers the propagation of a shock wave from a less dense to a more dense fluid, the interface being initially corrugated, and examines the growth of irregularities of this interface. After establishing the differential equation of the problem, in which the unknown function is the pressure perturbation, author examines in detail the boundary and initial conditions. Boundary conditions are given successively at the reflected and transmitted shocks as well as at the interface. The initial conditions are then deduced. The problem to be solved is linear, but as considerably simpler problems led to very complicated solutions, the numerical integration was used for this case . For this purpose, the finite difference method is applied and the calculations are perfomed with a Los Alamos MANIAC computer. Results show that the corrugations of the interface are superstable as their oscillatory motion is damped.

T. D. Oroveanu, Roumania

4877. Ferrari, C., On transonic flow around airfoils with attached shock wave (in Italian), *Aerotecnica* **40**, 2, 94-98, Apr. 1960.

Conditions for existence of shock waves arriving at sonic line between subsonic and limited supersonic regions are studied. Paper is based on approximation by Tricomi and is related to previous research by P. Germain. Strong and weak shocks are studied. Strong shocks are found to be impossible within assumption of paper, whereas weak shocks are possible if lines corresponding to wave front in hodograph plane are tangent to a line parallel to coordinates. Local solution is then established, satisfying shock equations and being analytic continuation of solution for elliptical semiplane.

From author's summary by A. Balloffet, USA

4878. Slezkin, N. A., On the theory of gas flow in a layer between a shock wave surface and a blunt body of revolution (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 2, 3-12, 1959.

Author investigates the layer of the gas between the bow-shock in front of a blunt body and the body itself in a uniform incoming stream. The flow is steady. With the use of the continuity equation in the regions in front and behind the shock author constructs the formulas for the velocity components, Reynolds number and pressure in the layer behind the shock. Author mentions about three possible types of layer behind the shock: (a) Reynolds; (b) Oseen, (c) Prandtl.

In his particular case, author uses the classical Navier-Stokes equations, including the energy equation; this system is remodeled so as to reduce to a much simpler set of equations, called Reynolds equations. Since this system is nonlinear, author introduces the notion of some sort of a linearization or small perturbation functions for all the variables in question. The resultant linear system is remodeled once more by introducing dimensionless variables into the form called the Oseen form. In the next section author investigates the boundary conditions on both sides of the layer; these refer to the velocity components, their derivatives, pressure, density and temperature. In the last chapter author applies a "finite" (not infinitesimal) transition to the limit $x \longrightarrow 0$, i.e., to the stagnation point on the body. The limits of various composite functions are found by means of the l'Hospital rule.

The paper is a characteristic one for the present Russian organization of various branches of both pure and applied sciences. Published in the division of Mechanics and Machine Design (Izvestia) of the Russian Academy of Sciences, it contains only the derivation of the equations without any example whatsoever or numerical calculations.

M.Z.v. Krzywoblocki, USA

4879. Grozdovskii, G. L., Diukalov, A. N., Tokarev, V. V., and Tolstykh, A. I., Selfsimilar motions of gas with shock waves, spreading with constant speed into gas at rest, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **23**, 1, 281-286, 1959. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

This paper extends the work of Sedov, dealing with nonstationary selfsimilar motions of plane shock waves, to the axially symmetric case, i.e., the selfsimilar motions of radially converging and diverging shock waves. The relationship to steady axially symmetric flows is indicated.

M. Epstein, USA

4880. Huber, P. W., and McFarland, D. R., Effect of surface roughness on characteristics of spherical shock waves, NASA TR R-23, 25 pp., 1959.

Small-scale detonations over both smooth and rough plates were used in study of shock peak overpressures and Mach stem heights. Roughness (distributed pyramid elements) reduced surface overpressures slightly and lowered stem heights. An earlier suggestion of a geometrical (triple point) similarity parameter was not applicable and an empirical alternative is offered. Data were based on schlieren photographs and air flow details are absent.

J. R. Baron, USA

4881. Bartlma, F., Calculation of spherical shock waves by the method of characteristics (in German), Dtsch. Versuchsanstalt Luftfahrt, Ber. 103, 34 pp., Dec. 1959.

4882. Hurlbut, F. C., Influence of pressure history on momentum transfer in rarefied gas flows, *Physics of Fluids* **3**, 4, 541-544, July/Aug. 1960.

The coefficient for tangential momentum transfer σ between the boundary and a flow of rarefied gas has been investigated by the rotating cylinder technique. Systematic perturbation of the pressure history of the active surfaces was made and correlated with

observed variation of σ. A high-pressure history is found to result in a lowering of σ with time (order of days) from values in the range 0.9 to 1.0 values approaching 0.6. A low-pressure history is found to result in constant values of σ in the range 0.9 to 1.0. The observations were of the same character for all gases.

From author's summary by H. Mirels, USA

4883. Ziering, S., Shear and heat flow for Maxwellian molecules, *Physics of Fluids* **3**, 4, 503–509, July/Aug. 1960.

In previous studies treating the problems of linear shear and heat flow in parallel plate geometry, a half-range moment expansion of the distribution function was given for arbitrary values of the Knudsen number. The half-range collision integrals for Maxwellian molecules are now evaluated numerically. The inverse fifth law of molecular interaction can be regarded as a lower bound for neutral gases. The problems of shear and heat flow are reconsidered in this light.

From author's summary by H. Mirels, USA

4884. Badal'yan, R. G., An approximate integration of the equation for the unsteady one-dimensional motion of a gas (in Russian), *Izv. Krymsk. Ped. In-ta* **29**, 269–282, 1957 (1958); *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11340.

An investigation is made of the nonlinear equation for the one-dimensional adiabatic motion of a gas expressed in Lagrangian variables. In order to find the solution the adiabate is replaced by a curve of such a nature that the initial solution in the final computation merged either with the system of linear equations or with a system of equations in which, as a minimum, one of the families of characteristics is rectilinear. The solutions obtained are applied to the investigations of the motion of a gas in an infinite tube, in a tube with one closed end and the motion of a gas between the piston and the wall moving in accordance with the given principle.

M. P. Mikhailova

Courtesy Referativnyi Zhurnal, USSR

4885. Tsynkova, O. E., On auto-oscillations in supersonic diffusers (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 5, 19–26, 1959.

The problem of auto-oscillations in supersonic diffusers is solved by the method of straight (linear) "running" waves.

Analysis of the general solution allows to select results reflecting the substance of the phenomenon while rejecting those caused by the effects of linearization.

The criterion of stability is obtained directly from the solution.

From author's summary by A. N. Petroff, USA

4886. Greer, H., Rapid method for plug nozzle design, *ARS J.* **31**, 4, 560–561 (Tech. Notes), Apr. 1961.

An approximate method is presented for determining the contour of a plug nozzle for a given design expansion ratio and ratio of specific heats. This procedure is based on centered isentropic wave expansion and geometric considerations.

From author's summary

Boundary Layer

(See also Revs. 4814, 4842, 4848, 4878, 4885, 4901, 4951 4956, 4976, 4982, 4990, 4998, 5002)

4887. Abbott, D. E., and Kline, S. J., Simple methods for classification and construction of similarity solutions of partial differential equations, AFOSR TN 60–1163 (Stanford Univ., Dept. Mech. Engng. Rep. no. MD–6), 48 pp., Oct. 1960.

A similarity solution of a partial differential equation utilizes a combination of the original independent variable as a new variable in such a way that the number of independent variables is reduced. The present investigation describes two methods for determining such combinations of variables. A number of illustrative problems are discussed, mostly involving laminar flows.

Reviewer considers this an interesting presentation of these methods but is surprised that the bibliography does not refer to the extensive Russian literature. For example, a book by Sedov [AMR **13**, (1960), Rev. 3782] gives numerous gas dynamical applications; also, the well-known book "Hydrodynamics" by Birkhoff contains an illuminating discussion.

W. Squire, USA

4888. Limberg, H., Laminar entrance flow in a convergent channel (in Roumanian), *Studii Si Cercetari Mecan. Appl.* **12**, 1, 47–59, 1961.

Author considers the laminar incompressible two-dimensional entrance flow in a convergent channel. The flow consists of the boundary layers which develop on the two walls and a central flow. The particularity of the problem treated lies in the determination of the central flow velocity, at the outer edge of the boundary layer. For this purpose, the condition of the conservation of mass flow is used. The mathematical treatment is based upon the "Gortler new series," which is advantageous in that it is rapidly convergent. Author's laborious and accurate calculations lead to the determination of the additional terms of the central flow velocity, the principal term being that of the point-like source velocity.

St. Săvulescu, Roumania

4889. Becker, E., The laminar incompressible boundary layer along a plane wall deformed by harmonic waves (in German), *Z. Flugwiss.* **8**, 10/11, 308–316, Oct./Nov. 1960.

A plane surface is subjected to harmonic oscillations causing wave motion in the direction of a two-dimensional laminar incompressible boundary-layer flow. The resulting nonsteady flow is examined by linear approximation.

A. R. Mitchell, Scotland

4890. Helke, G., Tables of universal functions for the calculation of axially symmetrical laminar boundary layers (in German), Dtsch. Versuchsanstalt Luftfahrt, Ber. no. 135, 32 pp., Sept. 1960.

This report contains tables of the coefficient functions for the Görtler series for the calculation of laminar boundary-layer flow of incompressible fluids around solid bodies of revolution with the stagnation point in front (value of the parameter $\beta_0 = 1/2$).

From author's summary by S. I. Pai, USA

4891. Acrivos, A., A rapid method for estimating the shear stress and the separation point in laminar incompressible boundary-layer flows, *J. Aero/Space Sci.* **27**, 4, 314–315 (Readers' Forum), Apr. 1960.

4892. Akatnov, N. I., The application of von Mises' variables to the problem of laminar boundary layer propagation along a wall, *Appl. Math. Mech. (Prikl. Mat. Mekh.)* **24**, 1, 208–212, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N. Y.)

Author considers two-dimensional case of source located at the leading edge of a semi-infinite flat plate wherein the source flow is superposed on a flow, $U_0 = \text{const}$, in the direction of the plate. The incompressible boundary-layer equations are transformed to the well-known von Mises variables, and solutions are sought for the laminar boundary layer along the plate. For case of $U_0 = 0$, the velocity profiles are determined and it is found that the wall shear stress due to source alone varies as $\tau_w \sim x^{-5/4}$. For $U_0 \neq 0$, solutions are found which are valid far downstream of the leading

edge. As distance downstream increases, solution tends to the Blasius solution. The effects of the additional velocity due to source flow can be determined. A. Ritter, USA

4893. Saljnikov, V., Extension of the Görtler series for the calculation of boundary layers about axially symmetric bodies (in German), Dtsch. Versuchsanstalt Luftfahrt, Ber. no. 133, 82 pp., Oct. 1960.

By the help of Mangler transformation the series method developed by Görtler for two-dimensional laminar boundary-layer flow has been extended to the case of axisymmetric bodies of revolution at zero angle of attack and ring-shaped bodies of revolution. Numerical examples are given both for practical application and as a test of the accuracy by a finite series approximation.

Tables of the universal functions for the case $\beta_0 = 1/2$ are published in DVL Report 135 by G. Helke [See Rev. 4890 this issue] whereas tables for the case $\beta_0 = 0$ and $\beta_0 = 1$ have been published in the DVL Report no. 34 by Görtler.

From author's summary by S. I. Pai, USA

4894. Jonas, P., Integrals for stationary laminar flow with closed streamlines in approximated circular regions (in German), *Czech. J. Phys.* **10**, 10, 737–741, 1960.

For large Reynolds number ... the integral of the resistance force along the boundary flow region is directly proportional to the length of this boundary and to the magnitude of the velocity of rotation in the nucleus of the flow. The volumetric flow rate through the section of the recirculation boundary layer is directly proportional to the integral along the boundary of the region of the difference of the squares of the velocity in the boundary of the nucleus and the boundary of the region and inversely proportional to the magnitude of the velocity of rotation in the nucleus and the length of the boundary of the region.

From author's summary by W. L. Sibbitt, USA

4895. Bunimovich, A. I., On rarefied viscous gas flow past a plane semi-infinite plate (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 5, 16–18, 1959.

Author investigates the flow of a rarefied viscous fluid past a semi-infinite plate. The Prandtl number is assumed to be equal to unity, the coefficient of viscosity is proportional to the n-th power of the temperature and in the first step the author applies Dorodniczyn's (similar to Howarth's) transformation of coordinates (the so-called stretching of the y-coordinate). In the second step the author uses dimensionless coordinates, the notion of the streamfunction and preserves the type of the partial differential equations of both equation of momentum and energy. The boundary conditions are chosen in the form of those proposed by Knudsen and von Smoluchowski, i.e., the velocity at the surface of the plate is not zero but is proportional to the normal gradient of itself and the temperature has a jump at $y = 0$. The coefficients of proportionality contain the accommodation coefficient. The streamfunction and the temperature are developed in power series of small argument and the technique well known in the theory of boundary layer is applied: by means of comparing equal powers of the small argument, the system of two partial differential equations is associated with an infinite system of ordinary differential equations. The author calculates the zero-th and the first approximations to the velocity and temperature. Next, the friction coefficient is calculated in terms of the derivatives of the streamfunction at $y = 0$, wall temperature, viscosity coefficient and the coordinate x. Similarly, the heat transfer coefficient is calculated in terms of the above-cited quantities and Mach number. A numerical example, calculated at the end of the paper refers to $\gamma = 1.4$, $2 \leqq M_\infty \leqq 10$, $2 \leqq T_w/T_\infty \leqq 10$. A discussion of the results closes the paper. M. Z. v. Krzywoblocki, USA

4896. Petukhov, I. V., Integration of the boundary layer equations with the aid of an asymptotic solution (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **30**, 149–165, 1960.

A generalized coordinate transformation is derived for plane laminar boundary layers (and applied to flows near separation points with heat and mass exchange). The streamwise coordinate is subjected to the Dorodnitsyn-Stewartson stretching while the normal coordinate has in addition a $\tilde{y}_w(x)$ displacement which depends on initial (or upstream) conditions to make the asymptotic solution of the transformed problem near the outer edge of the boundary layer independent of upstream pressure history. A specific illustration is worked out. L. Trilling, USA

4897. Spence, D. A., Some applications of Crocco's integral for the turbulent boundary layer, Proc. Heat Trans. Fluid Mech. Inst., Stanford, Calif., June 15–17, 1960; Stanford Univ., 1960, 62–76.

Based upon results of experiments which indicated that the velocity profile in a supersonic compressible boundary layer is nearly a "universal" function of the Howarth-Dorodnitzyn variable author evaluates a form of Crocco's integral for turbulent Prandtl numbers different from unity to obtain the associated shear stress distribution, enthalpy distribution, recovery factor and heat-transfer coefficient. The second half of the paper generalizes the Stewartson-Illingworth transformation for flow with heat transfer. Particular choices of wall temperature distribution are found to lead to integrable forms of the integral momentum equation in the Stewartson variables when the recovery factor is not unity. Expressions are found for skin friction and the growth of the momentum thickness in a favorable pressure gradient with various rates of wall cooling. Paper is an extension of the work of Mager, AMR 8(1958), Rev. 564. R. L. Curl, USA

4898. Head, M. R., Entrainment in the turbulent boundary layer, Aero. Res. Counc. Lond. Rep. Mem. 3152, 17 pp., 1960.

Paper is based on the assumption that the rate of entrainment of fluid from outside into a turbulent boundary layer is a function of the shape of the velocity profile, the external velocity and the boundary-layer thickness, and is independent of the Reynolds number. This leads to an equation in the usual notation $(d/dx)[U(\delta - \delta^*)] = UF[(\delta - \delta^*)/\theta] = UF(H_1)$.

The universal function F is determined by experiment, and H_1 as a function of H is also found empirically. There is some scatter but it is possible to draw reasonable mean curves. The above equation, together with the usual momentum-integral equation, provides a new means of calculating the θ and H development. This is easy to apply and may be simply extended to cases of suction or blowing. Fair agreement with several different experiments is obtained. Finally the connection with the usual methods is elucidated. J. C. Cooke, England

4899. Khmeliuk, K. D., An investigation of the thermal boundary layer of vertical wall barriers (in Russian), *Inzbener.-Fiz. Zb.* **4**, 2, 109–111, Feb. 1961.

The nature of temperature oscillations, the zone of stable temperatures and distribution of velocities within the limits of free convection were investigated experimentally and obtained by means of microthermocouple and photographic recordings. The procedure is an aid to more precise determination of the coefficients of heat transfer.

From author's summary by A. N. Petroff, USA

4900. Motulevich, V. P., Heat transfer and friction of a plate in a gas flow during formation of a turbulent boundary layer with delivery of a foreign substance through pores (in Russian), *Inzbener.-Fiz. Zb.* **3**, 8, 31–38, Aug. 1960.

A method is presented for calculating friction and heat transfer of a flat porous plate, round which flows a stream of compressible

gas, when a substance distinct from the gas of the basic flow is delivered through pores, and on the surface of the plate is formed a turbulent boundary layer. This is a development of the work of Dorrance and Dor, who considered the analagous problem with a blast of homogeneous gas. Calculations using the proposed method are compared with experimental data.

From author's summary

4901. Powers, J. O., and Krahn, E., Heat transfer in dissociated air by a two-thickness integral method: Part 1, Stagnation point heat transfer, U. S. Nav. Ord. Lab. Rep. 6673, 41 pp., Sept. 1959.

A theoretical investigation of the compressible laminar boundary layer including the effects of dissociated air in equilibrium has been conducted by a two-thickness integral method. The method has been used for determining stagnation point heat-transfer and boundary-layer characteristics at pressures near one atmosphere and for wall and stream temperatures ranging from 400 deg R to 3,000 deg R and 600 deg R to 14,000 deg R, respectively. Comparison with the theory of Fay and Riddell indicates agreement for stream temperatures up to 10,000 deg R. Differences, which increase with increasing temperature, approach 30 per cent at a temperature of 14,000 deg R. The stagnation point results imply that the general application of the present method to blunt bodies of revolution is valid and also indicate areas of potential refinement. From authors' summary by R. D. Cess, USA

4902. Betz, E. A., On the effect of trip-wires on boundary-layer transition (in German), *Z. Flugwiss.* **9**, 1, 20–28, Jan. 1961.

Paper discusses the conditions under which a trip-wire is ineffective, partially effective, or fully effective as a turbulence trigger in the boundary layer on a flat plate. Partially effective means here that transition occurs upstream of the position of transition on the plate without wire but at a considerable distance downstream of the wire. The wire is called fully effective if transition occurs very closely behind the wire. Although the problem is defined by two independent Reynolds numbers (according to diameter of the wire and its position) only one Reynolds number is regionally relevant. Even in the case of a fully effective wire there is a finite distance between the wire and the position of transition. The Reynolds number formed with this distance and the free-stream velocity is of the order of 10^4. Also the effect of free-stream turbulence and pressure gradient is discussed.

J. C. Rotta, Germany

4903. Eppler, R., Results of general application in boundary layer and profile theory (in German), *Z. Flugwiss.* **8**, 9, 247–260, Sept. 1960.

Systematic calculations using a high-speed computer are carried out on airfoils with given pressure distributions to determine the effects of variations in angle of attack and Reynolds number on separation point location, instability of the laminar boundary layer, etc. A. R. Mitchell, Scotland

4904. Terrill, R. M., Laminar boundary-layer flow near separation with and without suction, *Phil. Trans. Roy. Soc. Lond.* (*A*) **253**, 1022, 55–100, Sept. 1960.

Author discusses the different approaches for studying the behavior of the laminar incompressible boundary layer without suction near separation point (papers by S. Goldstein, Stewartson, Hartree and Leigh). These solutions suggest that a singularity is present at stagnation point. In part I of this paper author extends the work of Goldstein and of Stewartson [AMR **12**(1959), Rev. 2501] to include suction through a porous surface. The analytic difficulties of this procedure are discussed in much detail. In part II a numerical method for solving laminar boundary-layer flow up to separation point is developed. It is based on a nonlinear ordinary differential equation for the velocity profile at station x_1, when the profile at x_1 is known. This nonlinear differential equation is solved by an iteration process which can be handled by modern computers. Examples treated: mainstream velocity $U = U_0 \sin x$ with constant suction and without suction. In addition author investigates a range of solutions of similar profiles which contains a separation profile for flow with suction or without suction. These profiles are compared to the profiles obtained by the numerical method. Irmgard Flugge-Lotz, USA

4905. Ovchinnikov, O. N., Longitudinal flow past a cylinder immersed in an inhomogeneous flow in the presence of steady boundary-layer suction, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **24**, 2, 541–545, 1960. (Pergamon Press, 122 E 55th St., New York 22, N. Y.)

A mathematical solution is given of the problem of the velocity and temperature distributions near a cylinder of radius $r = a$ placed lengthwise in the stream of a viscous incompressible fluid, the (undisturbed) velocity of which is given in the form $U = U_0 + \omega_0 r^2$ where U_0 and ω_0 are constants, in the presence of steady boundary layer suction, represented by a small negative component $v_r = v_0$ for $r = a$, vanishing for $r = \longrightarrow \infty$.

A solution of the Navier-Stokes equations is given, satisfying the above boundary conditions, and also $v_x = 0$ for $r = a$ and $v_x \longrightarrow U_0$ for $r \longrightarrow \infty$, v_x being the velocity component along the cylinder axis. From this, the friction shear stress at the wall is determined, giving for positive values the condition that the flow remains attached to the cylinder. It results that, for large Reynolds numbers, even a comparatively weak inhomogeneity of the oncoming stream will have a noticeable influence on the distribution of velocities and the shear stress at the wall.

The thermal boundary-layer on the cylinder is next investigated on the same basis using the heat-transfer equation; answer is also given to the thermometer problem and to the cooling problem.

Reviewer notes that some mistakes are found in the text, perhaps due to translation inaccuracies. In formula (1. 5) instead of R^k one should evidently read R^{-k}. In the title, "flow past a cylinder" should be corrected to "flow near a cylinder" as evidenced in the text.

As regards the originality of the contribution, reviewer is not aware that the peculiar problem has been treated elsewhere. The assumption of a constant radial velocity for the suction effect is in line with the most reliable treatments of such boundary-value problems. The choice of the peculiar velocity distribution law is evidently suggested by mathematical convenience; even so, the arbitrariness of the U_0 and ω_0 values permits covering a wide range of possible distributions. A. Ghetti, Italy

4906. Rheinboldt, W., On the calculation of steady boundary layers for continuous suction, with discontinuously variable suction velocity, NASA TT F-29, 99 pp., Mar. 1961.

4907. Chung, P. M., A simplified study of the nonequilibrium Couette and boundary-layer flows with air injection, NASA TN D-306, 69 pp., Feb. 1960.

Author presents a purposefully highly simplified study of effects of air injection and deviation from thermodynamic equilibrium on heat transfer in Couette flow and, by analogy with Rayleigh problem, on boundary layer over flat plate. Gas is taken to be a mixture of oxygen molecules, nitrogen molecules, and oxygen atoms, all fluid properties except temperature and atom mass fraction are held constant, and a linear reaction rate law is assumed (similar to Broadwell's model, *J. Fluid Mechanics*, **4**, Pt. 2, June 1958). The Couette flow problem is solved in closed form for all relaxation times and all degrees of surface catalyticity. The boundary-layer problem is solved for noncatalytic and

perfectly catalytic wall; variation in relaxation time is handled by an expansion in powers of the ratio flow characteristic time to relaxation time. The boundary-layer solution is thus a perturbation from a frozen flow solution. I. Greber, USA

4908. Nanda, R. S., Unsteady circulatory flow about a circular cylinder with suction, *Appl. Scient. Res.* (*A*) **9**, 2/3, 85–92, 1960.

Turbulence

(See also Revs. 4819, 4897, 4898, 4902, 4986, 5034, 5094, 5095, 5134)

4909. Loeffler, A. L., Jr., and Deissler, R. G., Decay of temperature fluctuations in homogeneous turbulence before the final period, *Inter. J. Heat Mass Transfer* **1**, 4, 312–324, Jan. 1961.

A previous analysis of homogeneous turbulence for times before the final period [Deissler, R. G., *Physics of Fluids* **1**, 111–121, 1958; AMR **12**(1959), Rev. 2510] has been extended to the case of temperature fluctuations in a homogeneous turbulence. The method consists essentially of the solution of the 2- and 3-point Fourier-transformed temperature equations after neglecting the fourth-order correlations in comparison with the second- and third-order correlations. Results are obtained for the convective transfer function, the spectral "energy" function and the total temperature fluctuation "energy." Comparison is made between the analysis and published experimental data obtained using air. The decay law obtained may be written $\overline{T^2} = A(t - t_0)^{-3/2} + B(t - t_0)^{-3}$ where $\overline{T^2}$ is the total "energy" (the mean square of the temperature fluctuations), t is the time and A and t_0 are constants determined by the initial conditions. The constant B depends on both initial conditions and the fluid Prandtl number. For large times the last term becomes negligible, leaving the $-3/2$ power decay law for the final period previously found by Corrsin.

It is shown that the effect of increasing Prandtl number is to extend the spectral "energy" function to larger wave numbers and to reduce the rate of decay of the temperature fluctuations.

From authors' summary by J. L. Lumley, USA

4910. Deissler, R. G., Analysis of multipoint-multitime correlations and diffusion in decaying homogeneous turbulence, NASA TR R-96, 30 pp., 1961.

Author develops first the two-point, two-time correlation equations and then extends the method also to the three-point, three-time case. He solves the final period by neglecting higher (third) order correlations and the earlier period by neglecting quadruple correlations. Such a strong assumption was used earlier for space correlations with no time difference by P. Y. Chou [*Quart. Appl. Math.* **3**, p. 38, 1945]. Approximate calculations of diffusion are also included.

Reviewer feels that the closure of the system of correlation equations by merely neglecting a certain higher-order correlation is not a very fruitful approach since the neglected correlations may prove to be quite significant. Some more subtle method (e.g. zero fourth cumulant, e.g. Proudman, I. and Reid, W. H., *Phil. Trans. Roy. Soc., Lond.* (*A*) **247**, p. 926, 1954) would be more satisfactory. L. S. G. Kovasznay, USA

4911. Saffman, P. G., On the effect of the molecular diffusivity in turbulent diffusion, *J. Fluid Mech.* **8**, 2, 273–283, June 1960.

A generalization of the well-known Lagrangian correlation between the velocity of a fluid particle at different times is presented. It is shown that the dispersion of a substance with a given molecular diffusivity in a stationary, homogeneous, turbulent velocity field can be formulated in terms of a "substance auto-correlation function." Also it is shown that the interaction between molecular diffusion and the turbulent motion reduces the dispersion from the value it would have if the processes of molecular and turbulent diffusion were independent and additive. The ratio of the contributions to the dispersion from the interaction term and the turbulent diffusion term is obtained for comparatively large times and is found to be inversely proportional to Prandtl number and Reynolds number of the turbulence.

From the author's summary by N. Z. Azer, Egypt

4912. Aris, R., On the dispersion of a solute in pulsating flow through a tube, *Proc. Roy. Soc. Lond.* (*A*) **259**, 1298, 370–376, Dec. 1960.

In previous papers [Aris, *Proc. Roy. Soc. Lond.* (*A*) **235**, p. 67, 1956 (AMR **9**(1956), Rev. 3128); **252**, p. 536, 1959] we have an alternative treatment of Taylor's analysis of simultaneous convection and diffusion in flow through a tube [Taylor, *Proc. Roy. Soc. Lond.* (*A*) **219**, p. 186, 1953; **233**, p. 446, 1954; **224**, p. 473, 1954]. We.... consider.... pulsations by making the velocity at any point a periodic function of time... It is found that the Taylor diffusion coefficient contains terms proportional to the square of the amplitude of the pressure pulsations. S. Corrsin, USA

Aerodynamics

(See also Revs. 4842, 4855, 4860, 4862, 4871, 4872, 4873, 4877, 4878, 4879, 4885, 4901, 4907, 4954, 4955, 4976, 5071, 5075, 5076)

4913. Wannenburg, J. J., Lift of a two-dimensional cascade of aerofoils—simplified method of calculation based on Schlichting's theory, *Aircr. Engng.* **32**, 382, 350–355, Dec. 1960.

Schlichting's method is simplified by making further assumptions. Angle of zero lift is written as a sum of separate contributions of camber and thickness. Lift-curve slope is assumed to depend only on cascade geometry, neglecting effects of profile shape. Only three coefficients are retained in trigonometric expansion of vortex distribution and only two in source distribution. Boundary conditions are satisfied at three points on the chord. Angle of zero lift and lift-curve slope are then written in terms of functions depending only on cascade geometry. These functions were computed from Schlichting tables and are tabulated over a range of stagger angles and pitch/chord ratios.

Tabulated functions permit hand calculations of lift of given profile at 72 different cascade configurations in 8 hours. Numerical examples show good agreement with more exact calculations of Schlichting. C. P. Kentzer, USA

4914. Harvey, J. K., Some measurements on a yawed slender delta wing with leading-edge separation, Aero. Res. Counc. Lond. Rep. Mem. 3160, 28 pp., 1961.

The report describes tests made on a yawed slender delta wing from which the flow separated at the leading edges. Measurements of surface static pressure, total-head surveys and flow visualisation were performed to give the forces acting on the wing and to determine the physical nature of the flow.

The tests were made at subsonic speeds but were taken within the region of conical flow near the apex of the wing, thus providing basic information for both supersonic and subsonic flows.

From author's summary

4915. Bagley, J. A., and Beasley, J. A., The shapes and lift-dependent drags of some sweptback wings designed for $M_0 = 1.2$, Aero. Res. Counc. Lond. Curr. Pap. 512, 33 pp. + figs., 1960.

The camber and twist distributions needed to produce a constant spanwise C_L-distribution and certain linear chordwise load distributions have been calculated by linearized supersonic theory

at $M_0 = 1.2$ for a set of 34 thin sweptback wings. The wing planforms cover a range of aspect ratios from 2.0 to 3.5 and leading-edge sweep angles from 55° to 70°. Both leading and trailing edges are subsonic at the design Mach number, and the slenderness parameter $\beta s/l$ is between 0.19 and 0.40.

The lift-dependent vortex and wave drags associated with these loadings have also been calculated, and appear not to be excessive in almost all the cases considered.

From authors' summary

4916. Lehrian, Doris E., Vortex-lattice treatment of rectangular wings with oscillating control surfaces, Aero. Res. Counc. Lond. Rep. Mem. 3182, 24 pp., 1960.

The vortex-lattice method for simple harmonic motion of general frequency [Aero. Res. Counc. Lond. Rep. Mem. 2961] is used to calculate the derivatives for rectangular wings with oscillating constant-chord flaps. The discontinuous chordwise boundary condition associated with full-span flaps is replaced by a continuous equivalent downwash which is determined on the basis of two-dimensional oscillatory theory. In the particular case when the frequency tends to zero, the equivalent downwash is obtained on a distinct quasi-steady basis; stability derivatives are then evaluated by using an alternative form of the vortex-lattice method for low frequency [Aero. Res. Counc. Lond. Rep. Mem. 2922]. To allow for the spanwise discontinuity due to outboard flaps, a further adjustment is made to the boundary condition by the use of partial-span downwash factors.

Comparison of the stability derivatives with values obtained by the Multhopp-Garner method indicates that the present treatment for low frequency is satisfactory for full-span and outboard flaps on planforms of aspect ratio 2 and 4. For general frequencies, results for aspect ratio 2 with full-span flaps compare well with the values for lift and pitching-moment derivatives obtained by Lawrence and Gerber.

From author's summary

4917. Sirazetdinov, T. K., A wing of finite span in a nonstationary flow (in Russian), *Trudi Kazansk. Aviats. In-ta* **33/34**, 43–58, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11495.

This is an investigation of the transient motion of a wing of finite span with a curvilinear axis in an incompressible liquid. The small vibrations and deformations are imposed on the basic motion with a constant velocity. The problem is the determination of the circulation over the span of the wing. The full circulation Γ round the section of the wing is presented in the form of the sum of three items: the circulation taking into account the change in the angle of attack during vibrations and deformations; the circulation produced by the obliqueness of the flow; and the circulation taking into account the influence of the vortex wake, that is, the portions of the vortex system depending on the nonstationary nature of the motion. The two last items are presented in the form of integrals of functions containing Γ and the intensity of the vortices γ. The magnitudes of Γ and γ are interconnected by a differential equation. Thus a system of integrodifferential and differential equations is obtained for the determination of the distribution of circulation over the wing span. In the case of harmonic vibrations the equations for the determination of the circulation are merged with a system of linear algebraical equations.

M. M. Vasil'ev

Courtesy Referativnyi Zhurnal, USSR

4918. Corafoli, E., and Nastase, Adriana, Study of slender triangular wings with forced symmetry in supersonic flow (in French), *Rev. Mécan. Appl.* **3**, 4, 373–391, 1958.

The linearized theory of compressible flow is applied to determine the aerodynamic properties of a class of triangular lifting surfaces having a symmetric normal velocity distribution $w(x_1, x_2) = w(x_1, -x_2)$ given by

$$w(x_1, x_2) = \sum_{q=0}^{n-1} w_{n-q-1,q} x_1^{n-q-1} x_2^{q-}$$

where the planform is situated in the x_1x_2 plane and the x_1 axis is aligned with the direction of the undisturbed flow. The wing is described as possessing natural symmetry if only even powers of x_2 occur, and forced symmetry if only odd powers occur. It is necessary in the latter case to introduce a plus or minus value for $W_{n-q-1,q}$ depending on whether x_2 is positive or negative. For either case, the flow is a homogeneous one of order n, and the number of dimensions can be reduced from three to two by application of the conical flow transformation of Busemann. Solutions are given for the pressure distribution, lift, and drag on wings having either natural or forced symmetry with n equal to 2, 3, or 4.

J. R. Spreiter, USA

4919. Pinsker, W. J. G., Aileron control of small-aspect-ratio aircraft; in particular, delta aircraft, Aero. Res. Counc. Lond. Rep. Mem. 3188, 23 pp., 1960.

The aerodynamic and inertia characteristics associated with small-aspect-ratio wings are shown to affect aileron control, causing a tendency towards excessive aileron power combined with poor initial response. Simple formulas are given for the determination of the critical parameters and the effects of these on some aileron maneuvres are analyzed. Touchiness of lateral trimming is also expected to add to the handling difficulties of such aircraft.

Finally, design criteria are discussed for the attainment of optimum aileron control.

From author's summary

4920. Hacker, T., Stability of partially controlled motions of an aircraft, *J. Aerospace Sci.* **28**, 1, 15–26, 50, Jan. 1961.

Paper presents theory on stability of an aircarft for the case when pilot's controlling intervention is taken into account. The approach of the problem can be considered as an extension of the concept of "stability under constraint" as developed by Neumark. First part of report is devoted to mathematical basis for the solution of the problem. Practical applications for both uniform and nonuniform basic motion of the aircraft give a good illustration of results to be achieved with the method.

Appendix I shows the limits of applicability of theory, while in Appendix II some thoughts are developed on the estimation of the controllability of the various kinematic quantities by the pilot.

Author also indicates possible extension of the study.

J. Buhrman, Holland

4921. Kuhn, R. E., and Grunwald, K. J., Lateral stability and control characteristics of a four-propeller deflected-slipstream VTOL model including the effects of ground proximity, NASA TN D-444, 70 pp., Jan. 1961.

The investigation of the lateral-directional stability and control characteristics of a four-propeller deflected-slipstream VTOL model in the transition speed range was conducted in the 17-foot test section of the Langley 300-MPH 7- by 10-foot tunnel. A large fairing on top of the rear fuselage was needed to eliminate directional instability in the power-off flaps-retracted condition. Even with this fairing some instability at small sideslip angles remained for power-on conditions with low flap deflections. The configuration exhibited a high level of dihedral effect which, coupled with the directional instability, will probably produce an undesirable Dutch roll oscillation.

From authors' summary

4922. Reshchikova, A. A., Calculation of the vertical velocities of the air by the overloads of the airplanes (in Russian), *Trudi Tsentr. Aerol. Observ.* no. 24, 30–43, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11560.

The known formula for the calculation of the vertical velocities of wind gusts by overloads on the airplane $w_1 = a \Delta n$ is compared with the formula $w = b \Delta n + v_x$ obtained by A. S. Dubov [P. A. Vorontsov, A. S. Dubov, *Trudi Gl. Geofiz. Observ.* no. 51, p. 113, 1955]. Here a and b are the coefficients, depending on the aerodynamic characteristics of the airplane, v_x the vertical velocity of the center of gravity of the airplane, Δn the overload subdivided in portions g. For the airplane type IL-12 the coefficient b proves to be 1.48 times smaller than the coefficient a. Thus for narrow vertical flows with a clearly defined boundary where the airplane is practically unaffected by the gust and $v_x = 0$, Dubov's formula gives reduced velocities by comparison with the ordinary formula. Commencing however from the determined values for the velocity v_x, Dubov's formula gives larger velocities than the ordinary formula. Experimental data are furnished which were produced by the aid of an integrator of overloads, giving the possibility of measuring v_x and of working in synchronization with an accelerograph. The mean quadratic divergence between w_1 and w_2 was shown to be equal to 0.81 m/sec. Maximum divergence occurred at large vertical velocities. The author is of the opinion that Dubov's formula furnishes more satisfactory results than the widely applied formula $w_1 = a \Delta n$. L. A. Dikii

Courtesy Referativnyi Zhurnal, USSR

Vibration and Wave Motion in Fluids

(See also Revs. 4630, 4717, 4807, 4847, 4889, 5064, 5092, 5097, 5166)

4923. Marsh, H. W., Exact solution of wave scattering by irregular surfaces, *J. Acoust. Soc. Amer.* **33,** 3, 330–333, Mar. 1961.

The scattering of a plane, simple harmonic acoustic wave incident upon the irregular, one-dimensional surface separating two media having different propagation constants and impedances is studied. An exact solution is obtained by use of Wiener's generalized harmonic analysis. The solution is then extended to electromagnetic waves and general elastic waves, to nonplanar, nonharmonic sources, and to three-dimensional surfaces of separation. Various types of successive approximations suitable for high-speed computations may be constructed as appropriate to the particular problem at hand. R. Heller, USA

4924. Marsh, H. W., Schulkin, M., and Kneale, S. G., Scattering of underwater sound by the sea surface, *J. Acoust. Soc. Amer.* **33,** 3, 334–340, Mar. 1961.

A theoretical study is made of the scattering of sound in isothermal water by the ocean surface, using the scattering theory developed in the preceding paper [see review above] and the Neumann-Pierson oceanographic model of the sea. As an application, the sound propagation in a surface-bounded channel is investigated, and a graph is drawn which shows the theoretical sea surface scattering loss as a function of the product of rms wave height by sound frequency. R. Heller, USA

4925. Binnie, A. M., Self-induced waves in a conduit with corrugated walls: Part 1, Experiments with water in an open horizontal channel with vertically corrugated sides, *Proc. Roy. Soc. Lond.* (*A*) **259,** 1296, 18–27, Nov. 1960.

An investigation is described of the wave pattern resulting from the flow of water in an open channel with corrugated sides. The resulting waves, which appear to progress upstream, form at a specific stream velocity for each geometric arrangement, but this velocity does not appear to be uniquely related to the wave properties. The size and number of corrugations and the channel depth, width and velocity of flow are varied. The observed wavelength appears as multiples of the corrugation pitch. The observed wave period agrees well with the period calculated from the usual formula for waves in water of finite depth, but with the wave number replaced by the magnitude of the vector sum of the wave numbers corresponding to the corrugation pitch and cross channel oscillation. The author describes the observed phenomena in detail. H. G. Farmer, Jr., USA

4926. Sakurai, A., On the problem of a shock wave arriving at the edge of a gas, *Comm. Pure Appl. Math.* **13,** 3, 353–370, Aug. 1960.

Author considers the phenomenon which occurs when a shock wave propagates through a nonuniform medium of decreasing density and reaches the boundary where the density vanishes.

He divides the problem into two parts: the first part corresponds to the stage before the shock wave arrives at the boundary and the second part to the subsequent expansion of the gas into the vacuum. This phenomenon is related to the shallow-water theory of bores moving up a sloping beach.

Author gives the separate solutions of the first and the second stages and then gives a general discussion of results and conclusions for both stages. T. Riabokin, USA

4927. Kryazhev, F. I., Sound field of a first-order plate wave in a layer of water, *Soviet Phys.-Acoustics* **6,** 1, 60–70, July/Sept. 1960. (Translation of *Akust. Zh., USSR* **6,** 1, 65–76, Jan./Mar. 1960 by Amer. Inst. Phys., New York, N. Y.)

Observed phase and amplitude are summarized as a function of depth and range for two shallow bodies of water, for the lowest mode and wavelengths from two to one times the depth. For a Caspian Sea area of 11-meter depth, results agree with calculation for an attenuating fluid bottom with refractive index $1.4 + 0.4i$. For a Moscow reservoir about 2 meters deep, acoustic results and direct observation show the upper one-meter layer in the bottom to be highly compliant, lossy, and gassy, with the higher acoustic velocity at greater depth. P. Rudnick, USA

4928. Lieberman, P., and Brown, E. A., Pressure oscillations in a water-cooled nuclear reactor induced by water-hammer waves, *ASME Trans.* **82** D (*J. Basic Engng.*), 4, 901–911, Dec. 1960.

The water-hammer pressure oscillations of the system considered are treated in two parts. First, the pipe is considered independent of the plenum chamber by the application of open-ended pipe boundary conditions. The transmitted and reflected pressure waves at the open-ended pipe are considered with particular emphasis on the wave transmitted into the plenum chamber. The analysis of the pressure oscillations in the pipe follows the usual acoustic approximations. Frictional effects are included in the analysis. In order to obtain an estimate of the pressure distribution in the plenum chamber, the three-dimensional system is reduced to an equivalent one-dimensional system of varying cross section and analyzed by the acoustic approximation. The analytical results appear to be consistent with the experimental observations of the oscillations. L. A. Pipes, USA

4929. Klabukov, V. M., Water hammer in water conduits furnished with a surge tank with additional resistance (The theory and data of full-scale field experiments) (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Str-vo i Arkhitekt.* no. 2, 182–193, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11843.

The results of calculations by numerical and graphical methods of water hammer in pressure water conduits equipped with surge tanks with resistances are compared with the data derived from field experiments. It is noted that the results of the calculations are approximately the same as the experimental. A number of recommendations are made for simplifying the calculations (reduction in the number of calculation phases; disregarding the elasticity of the walls of the surge tank).

G. V. Aronovich
Courtesy Referativnyi Zhurnal, USSR

4930. Buniatyan, B. L., The modelling of the phenomenon of an unsteady waveless motion in a system "tunnel-surge tower" (in Russian), *Izv. Akad. Nauk ArmSSR, Ser. Tekhn. Nauk* **11**, 6, 23-30, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11852.

The problem given in the title is studied. The criteria of similarity are established and also the correlations between the scale multipliers (to cover the cases of the geometrically perfect and the distorted models). Finally the question is investigated regarding the modeling of water hammer in a pressure system containing a surge tank with resistance. It is noted that in this problem all the criteria of similarity can be satisfied and values obtained for the scale multipliers for all the geometrical and hydraulic parameters of the system by distorting the model along the length of the tunnel and by lowering the velocity of spreading of the agitation.

G. V. Aronovich
Courtesy Referativnyi Zhurnal, USSR

4931. Eremenko, E. V., Some data on an hydraulic jump with side eddies (in Russian), *Dop. Akad. Nauk URSR* no. 11, 1181-1185, 1959; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 1156.

Experimental data are published on the length of an hydraulic jump with lateral eddies; the equation for the jump is verified and the fact established that it agrees satisfactorily with the experiments when $\alpha_0 = 1.08$. The narrowing and the widening of the transit flow in relation to the length of the jump is analyzed.

From author's summary
Courtesy Referativnyi Zhurnal, USSR

4932. Krasnitskii, M. S., The hydraulic jump viewed as a mixing agent (in Russian), *Trudi Kievsk. In-ta Inzh. Vodn. Kh-va* no. 7, 179-183, 1957; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11876.

This investigation was carried out in a horizontal hydraulic tank 7 m long, 0.4 m high and 0.225 m wide, fitted with a shield for the production of different forms of jump. In the experiments a solution of potassium permanganate was used as the solution to be subjected to mixing. Basing his conclusions on the experimental material collected the author recorded that: (1) the most suitable type of jump for the mixer was the perfect hydraulic jump with a parameter for the kinetic state in the initial section of the jump $\Pi_4 = 10$; (2) the best mixing was obtained when the solution was introduced before the jump or at the commencement of the surface eddying; (3) the most intensive mixing occurred in the jump itself and ended at some distance from it. A description and calculations are furnished for a compact plan for an hydraulic jump mixer which ensures the complete mixing of the water with the added solution.

M. F. Skladnev
Courtesy *Referativnyi Zhurnal, USSR*

4933. Plakida, M., Investigation of a leading wave and its action on protective works of the nature of a vertical wall (in Russian), *Trudi Tsentr. Nauk-i. In-ta Ekon. i Ekspluat. Vodn. Transp.* no. 15, 5-34, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11759.

Results are given of laboratory investigations on a model of a breakwater which were carried out in the Central Scientific Research Institute for Economics and Exploitation of Water Transport in 1954-1956. On the basis of this research new formulas are proposed for the determination of evenly acting lateral and suspended wave pressures acting on a breakwater in the form of a vertical wall which would replace N. N. Dzhunkouskii's method of calculation, incorporated in standard GOST 3255-46. The following deductions were recorded. As the leading wave is unsymmetrical and has a very steep incurved face, the central height of the wave is raised above the undisturbed level to a magnitude greater than or equal to $1/3\ (2b_1)$, where $2b_1$ is the height of the wave, that is to say that the leading wave is close to a solitary wave in size. The correlation $2b_1 \leqslant 0.65\ h_1$ is suitable for the maximum height of the leading wave and the depth of the water basin in the zone of its propagation. An appreciable sub-surface transposition of liquid particles takes place, resulting in the trajectory taking the form of open loop-shaped curves. The velocities of propagation of a leading wave calculated by the Lagrange and Boussinesq formulas differ from the measured by 10 to 15%. It is proposed to adopt the Boussinesq formula. The maximum wave pressure on the wall emerges on the wall sooner than the splash can be observed, when the height of the level of the water above the undisturbed level is equal to $0.8\ (2b_1)$. At the time of splashing the pressure drops on an average to 43% of the maximum. Although some of the curves of the measured pressures rise steeply, to form peaks, in comparison with curves recommended for the calculation pressures, the author holds that these peaks can be disregarded since the vibrations in the structure caused by the impulses damp down rapidly. The proposal is made to look for the maximum lateral pressure on the wall by means of the N. N. Dzhunkovskii formula but when doing this to consider its being applied at a height of $1/3\ (2\ h_1)$ above the undisturbed level, basing this proviso on the theory of solitary waves and on the results of laboratory observations. Formulas are given for the lateral pressure at the foundation of the wall and for evenly acting lateral and suspended pressures both for the case of a stone revetment submerged at the bottom's level and for the case of a surface stone revetment. It is recorded that the given method of calculation is suitable if the critical depth is not less than $1.6\ (2\ h_1)$ while the slope of the sea bottom is not more than 1 : 20. The proposed formulas were used to verify the coefficient for the factor of safety to shear and overturning of some structures of the type of vertical walls which had been in use for many years (at Sochi, Tuapse, Genoa). The values for the coefficient, obtained on the basis of standard GOST 3255-46, were found to be somewhat reduced.

V. S. Voitsenya
Courtesy Referativnyi Zhurnal, USSR

4934. Manabe, D., Analytical studies on the waves generated in a shallow water of medium depth, *Mem. Fac. Engng., Kyushu Univ.* **18**, 3, 64-100, Mar. 1959.

4935. Cherkesov, L. V., On the development of surface waves excited by a periodic pressure, *Soviet Phys.-Doklady* **4**, 4, 776-778, Feb. 1960. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **127**, 4, 774-776, July/Aug. 1959 by Amer. Inst. Phys., Inc., New York, N. Y.)

Fluid Machinery

(See also Revs. 4813, 4816, 4825, 4826, 4831, 4913, 4948)

4936. Richter, W., Calculation of pressure distribution on plane blades with strongly arched thick profiles in the case of incompressible flow (in German), *Ing.-Arch.* **29**, 5, 351-372, Sept. 1960.

Paper gives an extension of H. Schlichting's theory [AMR 8(1955), Rev. 3482] to blades with high camber-chord and thick-

ness-chord ratios under condition of inviscid flow. An approximate method is presented which allows the partly numerical and partly graphical computation of pressure distribution with aid of calculating rule or machine only and which is said to be carried out in about 50 hours for one blade. Singularity method is used with vortex and source distribution along the mean camber line of the blade. These distributions are expanded in a power series, the free parameters of which are determined by a system of 8 linear equations. Method has been proved by several examples, one of them is presented in detail in an appendix for demonstration of the calculation steps. Comparison of pressure distribution on three different blades with results of a much more complicated method (said to require 200-hours) and with experiments are satisfactory. An application of the method for flow on an airfoil is shown for the Göttingen 387 airfoil with good agreement of lift coefficient and pressure distribution with experiments up to about 60 or 100 angles of attack.

F. W. Keune, Germany

4937. Fujie, K., A study on the flow in a centrifugal impeller with passages of circular-arc in meridional plane, *Bull. JSME* **4,** 13, 94-101, Feb. 1961.

The flow in an impeller is affected by the incidence angle. The impellers used in this experiment are of three types, two of which have constant passage height, but one is with an inducer and another is without, and the third has a constant sectional area all the way from inlet to outlet, but without any inducer.

The influence of incidence angle on the flow in the impeller has been measured by the kiel probe and the yaw-meter that rotate with the impeller. From this result, it has been recognized that the separation of the flow occurs at the triangle area that is bounded by the trailing face of blade and the shroud.

The pressure distribution on the blade face that has been measured in the experiments is compared with that of the two-dimensional theory. This theory is found to be in comparatively good agreement, except for the region near the leading edges of the blades, with the experimental results; that is, the pressure on the blade face is influenced by the main flow only, and it is hardly affected by the secondary flows that complicate the flow through the impeller. From author's summary

4938. Rusanova, E. I., Analysis of the results of measurements of the dynamic stresses in the blades of an axial compressor (in Russian), *Trudi Nauchno-Tekhn. O-va Sudostroit. Prom-sti* **8,** 1, 133-159, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12452.

In the equations for the transverse vibrations of a blade author takes into consideration the influence of the centrifugal forces, the aerodynamic reactions and the forces of the inelastic internal resistance. In solving the equations it is assumed that the form of the vibrations would be the same as in the absence of dissipation forces. Energetic concepts are used in the determination of the values of the vibrations' amplitudes. The influence is analyzed of the angle of attack of the blade on the amplitude of the vibrations and on the dynamic stresses. Results of the experiments are described; an axial eleven-stepped compressor was examined; revolutions 8200 min, air consumption 19kg/sec, blade material, steel of mark 2X13. The correspondence of the experimental and theoretical data is recorded.

Ya. G. Panovko

Courtesy Referativnyi Zhurnal, USSR

4939. Okeil, M. E., The design of homogeneous flow and wake adapted marine screw propellers, *Inter. Shipbldg. Prog.* **7,** 72, 323-342, Aug. 1960.

Author proposes new method for designing marine screw propellers, based on combining momentum and blade element theories, to yield pitch distribution for required thrust. Method covers both homogeneous flow and wake-adapted propellers. Numerical examples and comparison with experimental results are given.

No consideration is given to method of blade section design, nor to detailed study of cavitation characteristics of blade element. Design of blade section does not influence theory forming basis of this paper.

Author discusses existing design theories, including vortex theory, requiring extensive correction factors of dubious worth. To compare results of his proposed design method with application of vortex theory (with corrections applied) author tried three load distributions on wake-adapted propellers: (a) Van Manen optimum, (b) Lerbs optimum, (c) assumption that each radius is part of an optimally loaded homogeneous-flow propeller of same diameter and rpm.

Author concludes:

(1) His proposed design method gives results in good agreement with experiment, although mean pitch ratio may be slightly higher than that found by experiment.

(2) Number of blades has little or no effect on performance of screw propeller. No corrections should be made for effect of multiple blades. (Reviewer's note: This seems to be a major conclusion of paper.)

(3) Induced velocities at plane of propeller are, in most cases, greater than the conventionally considered one-half the final induced velocities, owing to pressure distribution around the blades.

(4) When designing a wake-adapted propeller according to a given load distribution, with the help of diagrams and equations given, an appropriate effective wake factor as shown in Appendix II should be used. If volume mean wake factor is used, propeller usually develops less thrust than that for which it is designed.

(5) If pitch at each radius of a wake-adapted propeller is calculated according to local speed at that radius, pitch distribution lies between Van Manen and Lerbs optimum.

A. C. Ingersoll, USA

4940. Nakonechny, B. V., Experimental performance of a six-bladed vertical axis propeller, David W. Taylor Mod. Basin Rep. 1446, 31 pp., Jan. 1961.

This report presents the results of experimental investigations of the performance of a 6-bladed vertical axis propeller (Voith-Schneider type). The investigations were conducted at different pitch ratios using modified cycloidal and sinusoidal blade motion. The tests were carried out at constant rpm of 600, 700, 800, and 900, respectively, for a range of speeds from zero to the speed of zero thrust. The results are presented in form of performance characteristic curves using new nondimensional coefficients of the $K - \lambda$ type adapted to vertical axis propellers.

From author's summary

Flow and Flight Test Techniques and Measurements

(See also Revs. 4836, 5026, 5041, 5043, 5070, 5071, 5150, 5151)

4941. Weber, P., Effect of displacement on current meter measurement in a closed rectangular section: Part 1, Laboratory investigation; Part 2, Power plant tests (in German), International Current Meter Group (East Kilbride, Glasgow), Rep. 2, 49 pp., 1961; Rep. 3, 24 pp., 1961.

Thirty five current meters were mounted on a grid in an 8.5-sq. ft section of a test canal in Zurich. Error of +8% was disclosed due to displacement and mutual interference of meters. Derived

correction factors were applied for tests of two power plants by the use of 45 current meters. S. Kolupaila, USA

4942. Winternitz, F. A. L., and McDonald, L. M., Displacement effects in penstock flow measurements by means of current meters: Part 1, Aerodynamic tests (in English), International Current Meter Group (East Kilbride, Glasgow), Rep. 4, 44 pp., 1961.

Tests were performed in a wind tunnel at various locations of 37 current meters mounted on 1 to 8 radii. Errors up to +1.5% were disclosed due to mutual interference of meters and the grid effect. S. Kolupaila, USA

4943. Muller, H. P., The effect of oil viscosity on current meters during calibration and field tests (in English), International Current Meter Group (East Kilbride, Glasgow), Rep. 5, 4 pp., 1961.

Thirty four current meters were calibrated at 13°C and applied in a penstock at 31°C. Selection of proper oil of low viscosity assured required accuracy. S. Kolupaila, USA

4944. Muller, H. P., Experience regarding the number of current meters used for flow measurements in penstocks (in English), International Current Meter Group (East Kilbride, Glasgow), Rep. 6, 12 pp., 1961.

Swiss rules for turbine testing (1957) require a substantially larger number of current meters to be installed than the German standard DIN of 1957. Tests with different number of meters proved that this can be reasonably reduced under favorable local conditions. S. Kolupaila, USA

4945. Coffin, J., and Bertholet, G., Magnetic drive for current meters (in French), International Current Meter Group (East Kilbride, Glasgow), Rep. 7, 10 pp., 1961.

A water-tight box with electric contact transmits signals by means of magnetic drive. Advantages of this arrangement are explained. S. Kolupaila, USA

4946. Overberg, R. J., and Leaderman, H., Viscoelastometer for measurement of flow and elastic recovery, *J. Res., Nat. Bur. Stands.* **65** C (*Engng. Instrumentation*), 1, 9–21, Jan/Mar. 1961.

An account is given of the design, performance, and method of operation of a concentric cylinder rotational viscoelastometer. The instrument is designed for measurement of flow and time-dependent elastic recovery of viscous elastic liquids for temperatures at which the viscosity lies in the range of 10^6 to 10^{12} poises and the elastic recovery is to be studied over a range of time from 1 second to 1 day. It is also suitable for measurement of non-Newtonian flow when the apparent viscosity is not much less than the Newtonian viscosity. The stator is a 1.5-inch precision-bore glass tube, which permits visual observation of the sample. Interchangeable rotors (radius ratios 0.21, 0.58, 0.92) are provided, the smallest for materials which are only slightly non-Newtonian and have a small compliance. Torque is applied by weights through a torque drum. Mirrors on the rotor shaft and a photoelectric recorder permit continuous recording of the rotor position. Mercury slip rings permit monitoring of rotor temperature. The maximum torque is 5×10^7 dyne-cm, while the torque due to static friction is 2.5×10^4 dyne-cm. The maximum surface shear stress is 6.5×10^6 dyne-cm^{-2}. Temperatures from −70 to +160°C have been realized. "End-effect" studies have been made. Instrument compliance, temperature gradient correction, and the effect of inertia and friction on elastic recovery measurements are discussed. Data reduction methods for nonlinear behavior are developed. From authors' summary

4947. Young, L., and Wilson, G., Developments in salt-velocity flow measurement (in English), *Houille Blanche* **15**, 5, 569–600, Sept./Oct. 1960.

An extensive series of laboratory experiments, supplemented by field tests, has been made on the salt-velocity method of flow measurement. Comparative flow measurements were made by a standard weighbridge in the laboratory, and by current meters in the field tests. Novel apparatus has been constructed which is small and light for easy transport in field work, which will give satisfactory results in water of high initial conductivity, and which can be operated in pipe lines under high heads. Rapid methods of analyzing the observations have been developed, and as a result it has been found possible to design completely battery-operated detection equipment which should give virtually instantaneous measurement of flow.

The laboratory experiments showed that, subject to certain limitations, the apparatus gave results with 95 per cent confidence limits of error of ±1 per cent. The field tests appear to confirm this. From authors' summary

4948. Rudinger, G., A shock-tube technique to determine steady-flow losses of orifices and other duct elements, *ASME Trans.* **82** D (*J. Basic Engng.*), 1, 195–200, Mar. 1960.

It is shown that a simple shock tube is capable of producing appreciable steady-flow rates through a short duct element, such as an orifice, a valve, or a screen. The flow upstream and downstream of the test element and, therefore, also the losses caused by the test element, can be calculated from known initial conditions in the shock time and pressure measurements at one point upstream of the element. Experiments to determine the discharge coefficient of a sharp-edged orifice are described as an illustration of the method. The results are in good agreement with available steady-flow data.

From author's summary by A. F. Chapman, USA

4949. Broglio, L., A method to investigate kinetic heating at very high velocities in an intermittent wind tunnel (in Italian), *Aerotecnica* **40**, 5, 259–262, Oct. 1960.

An article on the application of the theoretical-experimental method of the author ("ICAS Meeting," Madrid 1958) in the open hypersonic wind tunnel of the University of Rome. The equipment to maintain a given surface temperature distribution during the experiment is described as well as the equipment to measure this temperature distribution and the local heat flux. Preliminary results are given for the recovery factor and convection coefficient on a 20° cone, with a rounded nose of radius $r = 15$ mm at a Mach number of 4. The flow becomes turbulent at $x/r = 0.84$ where x is the curvilinear distance measured along the surface from the stagnation point. E. A. Brun, France

4950 Amick, J. L., Pressure measurements on sharp and blunt 5° and 15°-half-angle cones at Mach number 3.86 and angles of attack to 100°, NASA TN D-753, 45 pp., Feb. 1961.

Measured pressure distributions on cones are compared with modified Newtonian theory. Deviations as large as 14% of the stagnation pressure behind a normal shock are found. By combining empirical results for cylinders normal to the flow with Newtonian concepts, a method of claculating pressures on cones at high angles of attack is developed. Calculations by this method differ from the experimental results on sharp cones by only 2% of the stagnation pressure behind a normal shock. For blunted cones, additional deviations up to 8% are noted near the nose. From author's summary

4951. Daiber, J. W., An optical boundary-layer probe, *J. Aerospace Sci.* **27**, 11, 836–840, Nov. 1960.

Schlieren method combined with a photomultiplier tube is used for measuring the density distribution in the laminar boundary layer behind a traveling shock in a shock tube. The method is based on the fact that the output of the photomultiplier tube placed just behind the focal point of the schlieren system is proportional to the integral of the density derivative, as described by Resler and Scheibe [AMR **10**(1957), Rev. 1509]. Density profile in the boundary layer at a shock Mach number of 1.58 is compared with the theory of Mirels [AMR **9**(1956), Rev. 206]. Experimental curve is considerably displaced from theoretical one due to the error in determining the position of the lower edge of the light beam above the wall, but this discrepancy can be eliminated by adjusting one point to the theoretical curve. Author shows also that the photomultiplier-output-versus-time curve can be used to detect the transition from laminar to turbulent flow. F. Tamaki, Japan

4952. Yaggy, P. F., and Mort, K. W., A wind-tunnel investigation of a 4-foot-diameter ducted fan mounted on the tip of a semispan wing, NASA TN D-776, 31 pp., Mar. 1961.

Variations of duct angle, power required, and pitching moment were obtained for force-balanced conditions simulating level, unaccelerated, transition flight from 0 to 140 knots. The investigation included tests to determine the effectiveness of movable vanes in the duct. From authors' summary

4953. Tolhurst, W. H., Jr., Hickey, D. H., and Aoyagi, K., Large-scale wind-tunnel tests of exhaust ingestion due to thrust reversal on a four-engine jet transport during ground roll, NASA TN D-686, 26 pp., Jan. 1961.

The data presented include the free-stream dynamic pressure at the occurrence of exhaust gas ingestion in the outboard engine and the increment of drag due to thrust reversal for various modifications to both cascade-type and target-type thrust reversers. A motion-picture film supplement is available.

From authors' summary

4954. Jernell, L. S., Investigation of the static longitudinal and lateral stability characteristics of a 0.10-scale model of a three-stage configuration of the Scout research vehicle, NASA TN D-711, 27 pp., Mar. 1961.

An investigation was made in the Langley Unitary Plan wind tunnel to determine the effects of fin area and the effects of antennas and wiring tunnels on the static longitudinal and lateral stability of a 0.10-scale model of a three-stage configuration of the Scout vehicle. The tests were performed at Mach numbers of 2.29, 2.96, 3.96, and 4.65 and at Reynolds numbers of about 3.5×10^6 per foot. The results are presented without analysis.

From author's summary

4955. Turner, T. R., Ground influence on a model airfoil with a jet-augmented flap as determined by two techniques, NASA TN D-658, 18 pp., Feb. 1961.

An investigation was made in the Langley 300-MPH 7- by 10-foot tunnel with a conventional ground-board setup and in the Langley tank no. 1 by using the tow carriage to move the model over a ground board to evaluate the simulation of flight conditions in ground influence with a conventional ground-board setup. The 12-percent-thick airfoil was unswept and untapered with an aspect ratio of 6.0 and had a 10-percent-chord jet-augmented flap.

From this investigation it appears that the loss in lift of an airfoil with a jet-augmented flap in ground influence as determined in a wind tunnel with a conventional ground-board setup is considerably larger than would be obtained in free flight.

From author's summary

4956. Roshko, A., On flow duration in low-pressure shock tubes, *Physics of Fluids* **3,** 6, 835-842, Nov./Dec. 1960.

The present paper presents a logical analytic resolution of the "anomolous" low density behavior of shock tube performance first reported by Duff [*Physics of Fluids* **2,** p. 207, 1959]. Author confirms Duff's qualitative explanation of the large decrease in distance between the shock front and the driver gas front at low densities in terms of viscous flow through the contact surface. A similarity solution is found for the variation of this distance with time, with the solution depending implicitly on the form of the boundary layer after the shock, this being estimated both empirically and in approximate analytic terms. The maximum flow duration behind the shock is found to increase linearly with pressure and quadratically with tube radius and to decrease strongly with increasing shock Mach number.

Reviewer considers this paper to be a definitive work that eliminates any arguments that may remain against Duff's conclusions. J. A. Laurmann, USA

Thermodynamics

(See also Revs. 4853, 4989, 5024, 5036)

4957. Koeppe, W., Contribution to the Joule-Thomson effect (in German), *Monatsber. Dtsch. Akad. Wissenschaft. Berlin* **2,** 2, 71-78, 1960.

An integration of the Joule-Thomson coefficient over finite pressure intervals is carried out for gases and gas mixtures in which there is no condensation. Throughout, the gas properties are represented by the virial coefficients. Since molecular theory provides adequate data for the second coefficient and its temperature derivative, but not for the higher coefficients, these higher coefficients are disregarded, so that the resulting theory is a first- and second-order approximation. Tests on numerous gases and gas mixtures—in some cases in dependence on relative concentration—and over wide temperature ranges, reveal very good agreement, however. This theory should thus be useful in assessing the temperature changes to be anticipated in an adiabatic throttle with finite pressure drop.

W. Hitschfeld, Canada

4958. Snider, R. F., and Curtiss, C. F., Kinetic theory of moderately dense gases: rigid sphere limit, *Physics of Fluids* **3,** 6, 903-904, Nov./Dec. 1960.

Solutions to the modified Boltzmann equation derived by Bogoliubov, which includes first-order density corrections, were given by the authors in an earlier paper [*Physics of Fluids* **1,** 122-138, 1958]. However, the coefficients of thermal conductivity and viscosity were evaluated improperly there for the limiting case of rigid sphere molecular interactions. The present paper corrects this part of the derivation and finds the comforting result that in the limit the solutions do approach the classic results of Enskog. C. F. Hansen, USA

4959. Potter, J. H., and Levy, M. J., The free expansion of dry and moist air, *ASME Trans.* **83** B (*J. Engng. Industry*), 1, 97-100, Feb. 1961.

If a gas is permitted to expand freely from a vessel of constant volume, the gas remaining in the vessel is generally considered to undergo reversible adiabatic expansion. For such a case, the theoretical backgrounds are developed from the points of view of thermodynamics and gas dynamics.

Experiments are reported in which it is shown that, for expansions occurring over short time intervals, the temperature changes are essentially those predicted for reversible adiabatics. Some

speculations are offered as to the mechanisms obtaining in the slower expansions. From authors' summary

4960. Kalafati, D. D., Phase transition in supercritical region and the curve of inversion (in Russian), *Teploenergetika* no. 1, 72–78, Jan. 1961.

4961. Goglia, G. L., and Van Wylen, G. J., Experimental determination of limit of supersaturation of nitrogen vapor expanding in a nozzle, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 27–32, Feb. 1961.

The limit line of supersaturation of slightly superheated nitrogen vapor as it expands in a two-dimensional nozzle has been determined. The results indicate that the degrees of supersaturation vary from 12.3 F at a condensation temperature of 109 R to 3.5 F at 126 R. These results are in agreement with work of other investigations with nitrogen gas initially at room temperature.

From authors' summary

4962. Novikov, I. I., and Trelin, Yu. S., Velocity of sound at the phase equilibrium of fluid-vapor. Velocity of propagation of sound in the saturated vapors of carbon dioxide (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **2**, 112–115, 1960.

This paper describes the experimental equipment and the results obtained in measuring the velocity of sound in the saturated vapor of CO_2 in the temperature range of 5 to 31.01 °C at the ultrasonic frequency of 500 kc/sec. The results were obtained by means of an acoustic interferometer and are given in the form of a graph and table. The accuracy is within ±0.2%.

In addition, the velocity of sound in the saturated CO_2 has been calculated theoretically. The theoretical results are somewhat lower than the experimental ones mainly because the formula used for the velocity of sound approaches the saturation point from the two-phase condition while the experimental results were approaching the saturation point from the superheated condition of vapor. This difference was used to calculate the discontinuity in the velocity of sound across the saturation curve. It amounts to 11.5% at 5 °C and 26% at $t = 25$ °C. V. Chobotov, USA

4963. Sklyankin, A. A., and Strelkov, P. G., Reproducibility and precision of modern numerical values of the entropy and enthalpy of condensed phases at the standard temperature (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **2**, 100–111, 1960.

It is well-known that calorimetric measurements performed by different laboratories often show very disturbing discrepancies. Since high-precision calorimetry is a difficult art, the National Bureau of Standards (Ginnings, Furukawa, *J. Amer. Chem. Soc.* **65**, no. 3, 1953; Furukawa, McCoskey, King, *J. Res. NBS* **47**, no. 4, 1951) suggested the employment of several standard substances for over-all check-measurements; among these, the measurement of the specific and latent heats of pure benzoic acid was proposed. Measurements on such standard substances provide a convenient, ready-made, over-all assessment of the errors of two different calorimetric measurements on some other substance, without the need to analyze point-by-point every possible source of inaccuracy.

The present paper gives details of check-measurements performed on benzoic acid in VNIIM (Vsesoiuznyi, Nauchno-issledovatelskii Institut Metrologii—All-Union Research Institute for Metrology). The measurement was that of the specific heat of benzoic acid in the range 4 °K to 410 °K with reference to the International Temperature Scale. It is concluded that the best measurements available agree within 0.1–0.2% in the range of temperatures covered by the International Temperature Scale. Larger discrepancies have been discovered in the range 20 °K to 90 °K (at 24 °K, 70 °K and 85 °K) and these have been correlated with possible discrepancies in the temperature scales used in the U. S. A. and in the Soviet Union.

J. Kestin, USA

4964. Nowak, E. S., Grosh, R. J., and Liley, P. E., Smoothed pressure-volume-temperature data for water in the critical region derived from experimental measurements, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 14–26, Feb. 1961.

A graphical technique was used to derive pressure-volume-temperature data from previous American, Czechoslovakian, and Russian experimental measurements. The data were derived at close intervals throughout the region encompassed by pressures from 3000 to 4000 psia, specific volumes from 0.030 to 0.120 cu ft per lb, and temperatures from 690 to 752 F. The average deviation of experimental values of pressure from the derived values was less than one part in 10,000 in the region for specific volumes greater than the critical volume (0.0525 cu ft per lb) and was less than ten parts per 10,000 in the region for specific volumes less than the critical volume. These deviations are thought to be insignificant since they are entirely within the uncertainty of recent experimental measurements. In a previous paper, deviations of some 100 to 200 parts in 10,000 in pressure were shown to exist between the data of various steam tables. Since the data in the steam tables were derived from the same primary data used in this paper, one may conclude that these deviations greatly exceed the experimental uncertainty and must therefore be charged to previous interpolation methods and the previous paucity of data. One of the important results of this study was the establishment of the fact that there was excellent agreement between recent measurements by Kirillin, et al. and older measurements by Keyes and Smith and by Havlicek and Miskovsky.

From authors' summary

4965. Sokolov, S. N., Lapoushkin, S. A., and Kopilov, N.I., Thermal-physical properties of silicon in a liquid phase (in Russian), *Teploenergetika* **8**, 1, 79–81, Jan. 1961.

4966. Avdonin, V. I., and Novikov, I. I., Velocity of sound at the phase equilibrium of fluid-vapor. Velocity of sound in the saturated water-vapor (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **1**, 58–62, May/June 1960.

In this paper the authors present the description of the experimental apparatus used and the results obtained in measuring the velocity of sound in saturated water vapor over the temperature range of 50 to 250 °C (this corresponds to 0.1 to 40 atm pressure) within an accuracy of 1%. In an effort to avoid any dispersion effects the measurements were made at frequencies of 500 to 1000 cps. The basis for measurements was the method of standing waves in an acoustic tube (interferometer).

In addition, the velocity of sound and the corresponding adiabatic exponent were calculated theoretically. It appears the difference between the experimental and theoretical results is small, not exceeding 3–4%. A comparison is made with some previously established values by other authors, showing them to be in error. It is claimed that the range of measurements and the accuracy obtained is the highest that has been published.

V. Chobotov, USA

4967. Littman, H., and Davidson, B., Theoretical bound on the thermoelectric figure of merit from irreversible thermodynamics, *J. Appl. Phys.* **32**, 2, 217–219, Feb. 1961.

Using linear phenomenological laws and the Onsager reciprocal relations, authors show that positive semidefinite form of the rate of entropy production leads to a lower bound of zero and upper bound of infinity for the Joffe figure of merit of a thermoelectric generator. I. M. Krieger, USA

4968. Savino, J. M., and Ragsdale, R. G., Some temperature and pressure measurements in confined vortex fields, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 33–38, Feb. 1961.

Studies were conducted on vortex flow generated within two right circular cylinders by injecting air through longitudinal vanes forming the chamber. The length-to-diameter ratios were 0.107 and 0.50. Experimental end wall static pressure distributions, some total pressures, and total temperature data are presented. The most significant finding was the large radial variations in the total temperature; this is related to the Ranque-Hilsch effect. Also discussed is the relationship between the static wall pressures and the effective velocities in the vortex.

From authors' summary

Heat and Mass Transfer

(See also Revs. 4632, 4671, 4796, 4846, 4852, 4897, 4899, 4901, 4905, 4907, 4909, 4912, 4949, 4958, 4963, 5032, 5038, 5066, 5104, 5145)

4969. Anisnov, S. I., and Perelman, T. L., On the investigation of heat transfer in the presence of chemical conversions, *Inter. J. Heat Mass Transfer* **1**, 4, 269–272, Jan. 1961.

Authors consider problem of unsteady heat conduction between two parallel infinite planes $x = \pm l$, with heat source depending on temperature as exp $(-E^*/T)$, where T = absolute temperature and E^* = activation energy. Partial differential equation is

$$\frac{\partial T}{\partial t} = a\nabla^2 T + \frac{K_0}{c_p \rho} \exp\left(-\frac{E^*}{T}\right)$$

Boundary conditions are: $T(l,t) = T_c$, $\partial T(0,t)/\partial x = 0$, $T(x,0) = T_0$. Differential equation is converted into an integral equation, and a method of successive approximation is shown. Limiting solutions for large and for small $\tau = at/l^2$ are derived. It is shown that in the absence of "explosion" the temperature at each point will not exceed, at any time, the corresponding steady-state temperature.

M. Morduchow, USA

4970. Baum, V. A., Thermal models for heat-producing elements of nuclear reactors (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 176–187.

Paper describes a particular example of the construction of a thermal model for a specified fuel element in a nuclear reactor. The heating element was made of compressed, pulverized graphite, and the distributed sources of heat were created by an electric current producing Joule heat.

J. Kestin, USA

4971. Jordan, D. W., Method for calculating boundary value problems in heat conduction for the cylindrical cavity and the half space, by means of convolution integrals, *Brit. J. Appl. Phys.* **12**, 1, 14–19, Jan. 1961.

A method for obtaining approximate solutions to certain types of boundary-value problems in heat conduction by the use of convolution integrals is presented. An advantage of this technique is that a local temperature variation can be computed without resorting to a field computation such as the finite difference method. Surface temperatures are considered principally and specifically; a cylindrical cavity is considered.

T. Ishimoto, USA

4972. Boikov, G. P., and Korolenko, Y. A., The temperature field in bodies of elliptical section with internal heat generation (in Russian), *Inzbener.-Fiz. Zb.* **3**, 12, 78–80, Dec. 1960.

The temperature field of a beam of elliptical section having inner heat generation is considered. Calculating the relationship of second temperature derivatives along the coordinates to constants, formulas are obtained permitting calculation of temperature at any point on the transverse section of such a beam.

The proposed relationships are checked by the elementary balances method, and it is shown that they are extremely accurate for metallic bodies.

From authors' summary

4973. Paul, R., Solutions of concentration-dependent diffusion equation, *Physics of Fluids* **3**, 6, 905–907, Nov./Dec. 1960.

Paper deals with the measurement of diffusion coefficients D by counterdiffusion of two gases through a capillary connecting two large vessels. Existing theory is extended to allow for the variation of D with concentration, and some consideration is given to transients within the capillary. No reference is made to the work of Andrew, *Chem. Engng. Sci.* **4**, p. 269, 1950.

J. F. Davidson, England

4974. Ascher, M., Explicit solutions of the one-dimensional heat equation for a composite wall, *Math. Comput.* **14**, 72, 346–353, Oct. 1960.

Paper considers an explicit finite difference scheme to solve title problem. These possible choices for the time mesh Δt are discussed. The need for such a study arises since a uniform choice of Δt throughout the whole structure is not economical. For each case, convergence is proved. An example illustrates the theory. A comparison of computing time is provided.

Y. L. Luke, USA

4975. Gavrilova, R. I., and Prudnikov, A. P., A problem in the theory of heat conductivity (in Russian), *Inzbener.-Fiz. Zb.* **3**, 5, 136–137, May 1960.

The heat-conduction equation with a heat source is solved for a cylindrical annulus. The solution is found as a Dini-Bessel expansion.

From authors' summary

4976. Emblik, E., Freezing and thawing processes in refrigeration accumulators, *Sulzer Tech. Rev.* **41**, 3, 13–16, 1959.

This note presents equations which give the transient temperature at various locations of a plane body when the adiabatic wall temperature, film coefficient and radiant heat flux are predescribed functions of time. The equations contain memory coefficients which are defined by the configuration of the body under consideration. Coefficients that are suitable for short heating times of thermally thick materials are presented for an insulated flat plate, semi-infinite composite body, composite flat plate and a semi-infinite body.

From author's summary by J. G. Knudsen, USA

4977. Gonzales, J. I., Charts simplify calculation of coefficients for air-gap heat transfer, *Mach. Design* **33**, 3, 131–134, Feb. 1961.

4978. Tien, C., The extension of Couette flow solution to non-Newtonian fluid, *Canadian J. Chem. Engng.* **39**, 1, p. 45 (Note to the Editor), Feb. 1961.

The temperature distribution in the plane Couette flow of a non-Newtonian fluid with a power law relation between the stress and strain is shown to be the same as for a Newtonian fluid provided the Reynolds number is suitably defined.

W. Squire, USA

4979. Barrow, H., Convection heat transfer coefficients for turbulent flow between parallel plates with unequal heat fluxes, *Inter. J. Heat Mass Transfer* **1**, 4, 306–311, Jan. 1961.

This paper supplements the considerable knowledge existing for asymmetric heat transfer for turbulent flow in tubes. Heat-transfer coefficients are obtained for the boundaries using an extension of Mizushina's analogy between heat transfer and momentum transfer.

The working formulas presented are applicable to practical problems albeit care is required in view of the restrictive assumptions made relative to Prandtl and Reynolds numbers. The graphically presented results indicate the importance of the heat-flux ratio as a parameter, particularly in the case of nonuniform heating. The results suggest applicability to asymmetrical flow in annuli. The communication is preliminary to an experimental program designed to provide relevant data bearing on the general problem of asymmetrical heat transfer. F. G. Keyes, USA

4980. Pahor, S., and Strnad, J., A note on heat transfer in laminar flow through a gap, *Appl. Scient. Res. (A)* **10,** 1, 81–84, 1961.

The problem of heat transfer in laminar flow through a gap between two semi-infinite parallel plates at constant temperature was recently studied by Agrawal. He solved this problem with the use of infinite Fourier sine series and derived an expression for the local temperature profile and the local Nusselt number as a function of the distance along the gap. A detailed solution for Peclet number $Pe = 1$ is given. Far enough from the entrance of the gap the local temperature profile of the fluid is almost independent of its initial temperature. In this paper this limit temperature profile is expressed with the confluent hypergeometric function, and the corresponding Nusselt number as a function of Pe is calculated.

From authors' summary by W. Hitschfeld, Canada

4981. Barrow, H., A semi-theoretical solution of asymmetric heat-transfer in annular flow, *J. Mech. Engng. Sci.* **2,** 4, 331–336, Dec. 1960.

Paper concerns a semi-empirical analysis of turbulent heat transfer in fully developed annulus flow of fluids whose Prandtl number equals unity. The distribution of shear stress was evaluated on the premise that the radius of maximum velocity in turbulent flow was the same as in laminar flow. The distribution of heat flux was obtained by assuming the seventh-power law for the velocity profile and for the condition that heat transfer took place at the inner wall. This is the only wall condition considered in the paper. Using these distributions together with the observed fact that the bulk temperature of the fluid occured close to the radius of maximum velocity, author modifies the Reynolds analogy and derives an expression for the Nusselt number

$$Nu = 0.0346\ F_o Re^{3/4} \qquad (\text{for } Pr = 1)$$

when Re is the Reynolds number based on equivalent diameter and F_o is a factor depending only on the annulus radius ratio. When compared with the empirical equation recommended by McAdams, a fair agreement is observed. B. T. Chao, USA

4982. Nigam, S. D., and Agrawal, H. C., A variational principle for convection of heat: Parts 1 and 2, *J. Math. Mech.* **9,** 6, 869–883, Nov. 1960.

Authors extend variational principle of Biot to various cases of heat transfer due to forced convection. Numerical results are given for one-dimensional problems of fluid flowing between parallel walls with uniform or parabolic velocity profiles. Agreement with exact solutions is excellent.

Reviewer notes that method should be applicable to "similar" solutions, but will become difficult for genuinely two-dimensional flows. Such problems, however, are worthy of investigation by this method. N. Curle, England

4983. Petukhov, B. S., Genin, L. G., and Malter, V. L., Forced convection heat transfer in pipes with internal heat generation within the fluid flow (in Russian), *Inzhener.-Fiz. Zh.* **3,** 9, 3–9, Sept. 1960.

It is shown that calculations of heat transfer for a stabilized flow of fluid with constant physical properties and internal heat generation can be made by means of the usual formulas for heat transfer during flow without internal heat generation, if the coefficient of heat transfer is calculated by the difference between the temperature of the wall and the adiabatic temperature of the wall.

On the basis of modern data on the velocity and eddy diffusivity of momentum distribution obtained by Reichardt, numerical calculations of the adiabatic wall temperature are made for Reynolds modulus from 10^4 to 10^6 and Prandtl modulus from 0 to 100. Interpolation formulas for calculating the adiabatic wall temperature, approximating results of theoretical calculations with an accuracy of ±10%, are proposed. The results obtained for the adiabatic wall temperature are compared with the results of Poppendiek's calculations and are in good agreement with experimental data obtained by Muller. From authors' summary

4984. Yablonsky, V. S., Asaturyan, A. S., and Khizgilov, I. K., Turbulent diffusion in pipes (in Russian), *Inzhener.-Fiz. Zh.* **3,** 3, 117-122, Mar. 1960.

A method is proposed for predicting the turbulent diffusion coefficient applying the theory of local turbulence. This method is based on a number of experiments with pumping two fluids successively through the same pipeline.

From authors' summary

4985. Motylevich, V. P., Heat transfer at any point of obtuse bodies washed by a stream of liquid (in Russian), *Inzhener.-Fiz. Zh.* **3,** 5, 17–23, May 1960.

A method is presented for calculating heat transfer at any point in bodies washed by a stream of noncompressible liquid. The method is based on an analysis of processes in the central stream of the current. The relationship $Nu = f(Pe)$, obtained for bodies of various shape, is of the greatest practical interest. The results are in agreement with relationships obtained by other authors.

From author's summary

4986. Uryukov, B. A., Heat transfer in the turbulent boundary layer of a fluid (in Russian), *Zh. Prikl. Mekh. Tekh. Fiz.* **3,** 119–125, 1960.

A Reynolds analogy analysis with $Pr = 1$ is worked out, dividing the turbulent profile into a linear laminar sublayer and two logarithmic layers with different coefficients. By selecting these semi-empirically, qualitative agreement with experiment is obtained. L. Trilling, USA

4987. Adrianov, V. N., and Shorin, S. N., Investigation of the combined process of heat transfer in a combustion chamber (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 107–117.

As a basis of the analysis the authors take the conservation equations of mass, momentum and energy, together with the equation of state, the chemical rate equation and the concentration equations of all the reacting components. From this system of equations they determine a group of dimensionless parameters, and finally arrive at a functional relationship for heat transfer at a surface of a gas-fired combustion chamber.

The details of the experimental equipment used were discussed in an earlier paper of the authors; however, the reference to this work is not given. Sixty-seven data points were obtained with a clean and 46 with a chromium magnesite dust-covered tongue of flame. Experimental results are presented in two figures, and correlations are given for both the brick-lined part and the entire combustion chamber. The results indicate that the Reynolds number and not the Boltzmann number, based on the theoretical combustion temperature, is the appropriate parameter to be used in correlating heat transfer at the surface of a gas-fired combustion chamber.

For the convenience of the readers who are not familiar with the nomenclature used by Russian engineers, the Boltzmann number

represents the ratio of energy transport by convection to that by radiation. It is defined as $Bo = \rho c_p u/\sigma T^3$, where ρ is the density, c_p the specific heat at constant pressure, u the average velocity, σ the Stefan-Boltzmann constant, and T the absolute temperature.

R. Viskanta, USA

4988. Oguri, T., Determination of rate of heat transfer between the gases and the cylinder walls of spark-ignition engine, *Bull. Fac. Engng., Yokohama Nat. Univ.* **9,** 1–29, Mar. 1960.

Paper reports on interesting researches done by author to obtain a formula governing the heat exchange occurring between combustion gases and cylinder walls in spark-ignition engines. To this end a 6-hp horizontal, 4-stroke, single cylinder, water-cooled engine was loaded at 0/4 to 4/4 of its nominal output, the temperature oscillations on the gas-side face of its piston head and the corresponding pressures in the combustion chamber being registered on films by means of a well-conceived experimental equipment described in detail in the text.

Oscillograms so obtained and indicator cards show neat curves, which testify to the accuracy of recording. A complete calculation proves that, although temperatures were gathered not on the piston face but some 0.07 mm deeper, at the terminals of the thermocouple-wires, errors were practically negligible. The heat-exchange rates deduced from these curves were compared with values given by the well-known Eichelberg formula, which proved to correspond only in part with the experiment.

An effort was made by author to improve the formula obtained by Elser on diesel engines:

$$Nu = 6.5\sqrt{Pe}\ (1 + \Delta S/2\,c_p)$$

Using similarity and the energy balance author obtained the formula

$$Nu = 1.75\,Pe^{1/3}\,(1 + \Delta S/c_p)\,(2 + \cos(\theta - 20^\circ))$$

Notations are conventional, Nu and Pe are the corresponding Nusselt and Péclet numbers, S is the gas entropy, c_p the specific heat at constant pressure, θ the actual crank-shaft angle.

Comparison of both formulas with experimental results shows a very poor agreement. This is not surprising since use of only partial similarity generally gives misleading results. Only if all dominant parameters are correspondingly introduced in the formulas, may they lead to really useful results. Capital parameters such as geometry of the combustion chamber, method and intensity of wall cooling, ratios bore-stroke and fuel/air, ignition advance, turbulence and vorticity in the combustion chamber, etc. etc. etc. were not considered by author.

L. G. Hamburger, Roumania

4989. van der Burgh, J., Thermal convection at a melting benzene surface, *Appl. Scient. Res.* (*A*) **9,** 4, 293–296, 1960.

Convective heat transfer is usually described by the formula

$$Nu = C(GrPr)^{1/4}$$

where Nu is the Nusselt number averaged over the surface of the body, Gr equals Grashof number, Pr Prandtl number and C a constant. The value for C was determined experimentally by immersing a sphere of solid benzene in a large volume of liquid benzene of uniform temperature. The rate of heat transfer is determined by observing the rate of melting of the sphere. After corrections were made for the cold liquid produced by the melting, a value of 0.525 was found for C. The experimental data agreed with the theoretical curve in the range $10^6 < PrGr < 10^8$ except for points using small temperature difference between fluid and sphere. Explanations are stated for this deviation.

A. Fuhs, USA

4990. Singh, S. N., Heat transfer by laminar flow from a rotating sphere, *Appl. Scient. Res.* (*A*) **9,** 2/3, 197–205, 1960.

The heat-transfer problem for the flow of an incompressible viscous heat-conducting fluid, due to uniform rotation about a diameter of a sphere, which is kept at a constant temperature, has been solved with viscous dissipation included. Due to inflow at the poles the cooler liquid is drawn from infinity toward the rotating sphere and this causes a lowering of the temperature there. After flowing in the boundary layer of the sphere the liquid becomes heated up and causes a rise in temperature near the equator. Numerical results are given for the case of water ($Pr = 5$) and it is found that the isothermals are surfaces of revolution flattened at the poles and elongated near the equator. The thermal and velocity boundary layers turn out to be of the same order of magnitude.

From author's summary by M. Tribus, USA

4991. Leslie, F. M., and Martin, B. W., Laminar flow in the open thermosyphon, with special reference to small Prandtl numbers, *J. Mech. Engng. Sci.* **1,** 2, 184–193, Sept. 1959.

4992. Kazakevich, F. P., The effect of roughness on the aerodynamic resistance of pipe bundles subjected to transverse flow of gas (in Russian), *Teploenergetika* no. 1, 56–58, Jan. 1961.

4993. Zhukauskas, A. A., and Shlanchyauskas, A. A., Heat transfer and resistance of pipe bundles arranged in chessboard order in transversal flow of liquids (in Russian), *Teploenergetika* no. 2, 72–75, Feb. 1961.

4994. Zhukauskas, A., Makarevichyus, V., and Shlanchyauskas, A., The influence of the longitudinal spacing on the heat emission in a group of tubes arranged in chess-board fashion (in Russian), *Trudi Akad. Nauk LitSSR* **B 4(16),** 159–162, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11999.

Results are presented of the experimental investigation of the heat emission of a 19-row group of tubes arranged chess-board fashion $S_1 = 1.99\,d$, $S_2 = 0.99\,d$ and a 28-row group similarly arranged $S_1 = 2.12\,d$, $S_2 = 0.61\,d$, when being reached transversely by air and water. The results obtained are compared with the results of previous investigations of a group of tubes similarly arranged with $S_1 = 2.12\,d$ and $S_2 = 1.85\,d$. It was established that the heat emission increases with a decrease in the longitudinal spacing of tubes S_2.

From authors' summary

Courtesy Referativnyi Zhurnal, USSR

4995. Bernshtein, R. S., Pomerantsev, V. V., and Shagalova, S. L., The mechanism of the resistance and heat emission in groups of tubes (in Russian), Vopr. Aerodinamiki i Teploperedachi v Kotel'notopochn. Protsessakh, Moscow-Leningrad, Gosenergoizdat, 1958, 251–267; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12000.

Results are given of the experimental investigation of the hydrodynamics and heat emission of closely-spaced tube assemblies carried out in a wind tunnel in the range of Reynolds numbers for R from 100 to 50,000. The experiments were made with three groups of tubes arranged in corridor fashion with correlation of the spacing of $S_1/d = S_2/d = 1.13$, $S_1/d = 1.13$ and $S_2/d = 2.26$, $S_1/d = 2.26$ and $S_2/d = 1.13$ and also with two groups arranged chess-board fashion with $S_1/d = S_2/d = 1.2$ and $S_1/d = 2.26$, $S_2/d = 1.13$. The external diameter of the tubes was equal to $d = 28$mm. The distribution of the pressure on the perimeter of the tubes was studied with the aid of a cylinder with orifices 0.5 mm in diameter. The cylinder during measurements was rotated successively at 15° angles round its own axis. The heat emission was studied by the method of analogy between diffusion and heat exchange. To achieve this the tubes were made from a porous humidity-conducting material and were filled with water. The measurements essential for the determination of the coefficient of heat emission merged with the determination of the amount of water evaporated in a fixed interval of time and the temperatures of the dry and wet-

bulb thermometer up to and beyond the group. The distribution of the heat emission on the surface of the tubes in the group was studied with the aid of a special measuring cylinder in which the slot 8 mm in width had an independent water feed. The graphs obtained of the distribution of pressures and of the coefficients of heat emission enable the hydrodynamics of closely-shaped tube assemblies to be explained. The data for heat emission are worked into a form of relation $N = f(R)$, and in the case of stabilized rows of assemblies, arranged corridor-fashion with R number values of $R > 7000$, approximate the power principle of $N = 0.177\ R^{0.64}$ with an accuracy of ±5%. At small values for the R number the heat emission depends only very slightly on R, diminishing with the decrease of R.

L. G. Genin

Courtesy Referativnyi Zhurnal, USSR

4996. Gebhart, B., Transient natural convection from vertical elements, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 61–70, Feb. 1961.

Digital computer was used to evaluate integral equations for laminar flow. No significant temperature overshoot was calculated; results were essentially independent of Prandtl number.

J. G. Bartas, USA

4997. Eichhorn, R., The effect of mass transfer on free convection, *ASME Trans.* **82** C (*J. Heat Transfer*), 3, 260–263 (Tech. Briefs), Aug. 1960.

Free convection from a vertical flat plate with blowing or suction of a fluid with the same constant physical properties as the ambient fluid but at a different temperature has been evaluated for "power-law" variation of the normal velocity at the wall with distance along the plate. Such a variation then defines a specific "power-law" wall-temperature distribution with distance along the plate with which similarity solutions for the problem are obtained.

Blowing or suction greatly affects the heat-transfer rates but has only a slight effect on the skin friction.

E. V. Somers, USA

4998. Yih, C.-S., Thermal instability of viscous fluids, *Quart. Appl. Math.* **17**, 1, 25–42, Apr. 1959.

The stability of a viscous fluid subject to a vertical body force which is heated from below (negative axial temperature gradient) in an insulated vertical tube or between two insulated vertical planes is analyzed by the usual normal mode type of analysis. The principle of exchange of stabilities is shown to be valid for the problem and is used to simplify the study.

The curves of neutral stability differ from those ordinarily obtained in hydrodynamic stability problems in that the minimum critical Rayleigh number occurs at zero wave number and, hence, these curves have no lower branch.

The same problem in vertical channels was treated by the reviewer [Prandtl Anniversary Volume "50 Jahre Grenzschichtforschung, 1955 and NACA TN 3458, 1955] for two other sets of plate boundary conditions in addition to that in the present paper. The method used in reviewer's treatment consisted of finding the fully developed solutions to the given problem; hence, exact closed form results were obtained which yielded the stability criteria. The author of the present paper referenced only the first of the reviewer's papers and made several incorrect and general comments relating the two without discussing the apparent differences.

It appears that the author has confused the results of the reviewer's two papers. For example, it is stated in the subject paper that the reviewer in the first of his papers obtained the critical Grashof number associated with symmetric motion and that this is not the significant value; this number was obtained by the reviewer in his second paper for the case of uniform heat generation in the fluid; without the heat generation the lower (and significant) critical Grashof number (associated with antisymmetric disturbances) is given in the first of the reviewer's papers. This minimum critical value is identical to that presented in the present paper; the equations for determining this value, however, do differ because of the two methods used. Therefore, for the case of channel flow no new result was obtained in the present paper; the present paper is of interest because of its corroboration of the earlier result.

The stability criterion determined for the tube is new; neglecting the effect of tube rotation on the mean orientation of the fluid it is found that the rotation does not affect the onset of instability.

S. Ostrach, USA

4999. Guinn, G. R., Aerodynamic heating of plane bodies of low thermal diffusivity, *ARS J.* **31**, 1, 158–160, Jan. 1961.

Author presents an analysis and equations to predict rate of accretion or melting of ice on a tube maintained respectively below or above the melting point of water. Results have applications in refrigeration accumulators in systems where demand may be sometimes greater and sometimes smaller than the capacity. Equations presented give rate of ice accretion, rate of ice melting, effect of compressor efficiency, and equilibrium conditions. Equations involve the geometrical, physical properties and rate factors associated with system. No numerical values are assigned to individual parameters but results are presented graphically in terms of lumped parameters. Final graphs are useful for design purposes provided values of the lumped parameters are known. No numerical examples are given.

J. G. Knudsen, USA

5000. Marenov, A. E., Investigation of heat exchange in supersonic flow of air in tubes, *ARS J.* **30**, 11, 1055–1059 (Russian Suppl.) Nov. 1960.

This paper contains certain results of an experimental investigation aimed at the determination of the local coefficients of heat exchange at supersonic speeds of gas flow in a cylindrical tube of constant cross section. The coefficients of heat exchange are expressed in terms of the numbers Nu_x, referred to the length of the experimental portion of the tube. It is known that in the region of steady-state flow the coefficient of heat exchange remains constant among the tube. In the initial portion of a straight tube, the dynamic and thermal boundary layers grow and cause the flow of gas to become unstable, and the coefficients of heat exchange to vary along the initial portion of the tube. The papers published thus far contain no computation formulas for the coefficients of heat exchange in the initial portion of the tube at supersonic speeds.

From author's summary

5001. Tsiklauri, G. V., and Usanov, V. V., Heat transfer at high speeds in a pipe (in Russian), *Inzhener. Fiz. Zh.* **3**, 11, 48–51, Nov. 1960.

A solution is found for a differential equation describing one-dimensional motion of a gas in a cylindrical channel. It is shown that it is possible to use the hydrodynamic theory of heat exchange in the range $M = 1.5$–3.

A formula is derived with which parameters of flow in any section of a cylindrical channel, including the initial section, can be calculated.

Results of calculations with this formula are compared with experimental data, and are in good agreement.

From authors' summary

5002. Cresci, R. J., MacKenzie, D. A., and Libby, P. A., An investigation of laminar, transitional, and turbulent heat transfer on blunt-nosed bodies in hypersonic flow, *J. Aero/Space Sci.* **27**, 6, 401–414, June 1960.

The shrouded model technique was used to find the heat-transfer characteristics of a spherically capped cone at stagnation tempera-

tures in the range 1500 °R to 1700 °R. The heat-transfer rate was obtained from measurements of the surface temperature-time history of a partially insulated one-dimensional plug. The results are compared with Lees' theory for the laminar boundary layer, with a method developed by the authors for the transitional and fully developed turbulent boundary layer, and also with the flat-plate reference-enthalpy method for the turbulent boundary layer. The theories are in good agreement with experiment on the conical portion of the model, but about 20% too high on the spherical portion. The authors conclude that the reference-enthalpy method is more accurate than their alternative method for the turbulent boundary layer and simpler to use. L. Mack, USA

5003. Marenov, A. E., Investigation of heat transfer in supersonic flow of air in a pipe (in Russian), *Inzbener. Sbornik Akad. Nauk SSSR* **25**, 179–187, 1959.

Measurements of local heat-transfer coefficients for supersonic air flow in a constant diameter cylindrical pipe are given in terms of Nusselt numbers based on length of test pipe. The coefficient is constant in established flow; but in the inlet region, because of the growth of thermal and dynamic boundary layers, the coefficient varies. No adequate analytical formulation for heat-transfer coefficients in inlet zones is available. L. Trilling, USA

5004. Alimov, R. Z., Heat and mass transfer during cooling by evaporation of surfaces heated to a high temperature (in Russian), *Inzbener.-Fiz. Zb.* **3**, 5, 31–39, May 1960.

Criterial equations are deduced for the calculation of heat and mass transfer during cooling by evaporation of surfaces around which hot gases flow. Temperature and coolant liquid rate, determined by means of these equations, have been compared with experimental and theoretical data obtained by other authors.

From author's summary

5005. Polyak, G. L., Radiant heat transfer from bodies with arbitrary directional reflection from the surfaces (in Russian), Konvektivyni i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 118–132.

Author starts the analysis with the definition of the effective intensity of radiation which is defined as the sum of the intensity due to the emission from the surface plus the intensity due to the reflection of part of the radiation incident on the surface having arbitrary reflection characteristics. Since the effective intensity of radiation is in a form of an integral equation, an iterative type of solution of this equation is written in terms of well-known Neumann series. Diffuse reflection is then discussed and, as an example, a spherical annulus is considered. Heat-transfer equations are then derived for the cases: (a) both surfaces are diffuse reflectors, (b) the inner surface is a specular reflector and the outer surface is diffuse, (c) the outer surface is a specular reflector and the inner surface is diffuse, and (d) both surfaces are specular reflectors. No numerical results are presented.

Considering the importance of the problem for many radiant heat-transfer applications, the publication of the paper is welcome; however, the analysis is in most part a review of earlier published studies. R. Viskanta, USA

5006. Walker, R. E., and Grenieski, S. E., Jr., Instrument for measuring total incident radiant heat transfer to a jet engine surface, *ARS J.* **31**, 1, 77–78 (Tech. Notes), Jan. 1961.

5007. Allingham, W. D., and McEntire, J. A., Determination of boiling film coefficient for a heated horizontal tube in water-saturated wick material, *ASME Trans.* **83** C (*J. Heat Transfer*), 1, 71–76, Feb. 1961.

Authors study boiling of water at a surface in contact with a saturated wick made up glass fibers. Only one wick and one heat-transfer surface are studied so results are limited to particular system. Correlation is obtained on basis that significant parameter in the process is the capillary diameter at the surface. Laminar flow assumed in capillary and dimensionless correlation involving Prandtl number, pressure number and Reynolds number (based on equivalent diameter of capillary and porosity) satisfactorily correlated data obtained at the three different pressures and three different hot fluid temperatures. Authors point out that form of correlation is similar to laminar flow equation for heating in tubes, thus substantiating assumption of laminar flow in capillaries of wick. Reviewer believes there is little basis for their conclusion because of highly empirical nature of correlation presented. Rate of heat with wick varied as transfer differed to 0.3 power as contrasted to an exponent of 2.3 for nucleate pool boiling. Wick improves coefficient over pool boiling at low ΔT and is inferior at high ΔT. J. G. Knudsen, USA

5008. McFadden, P. W., and Grosh, R. J., An analysis of laminar film boiling with variable properties, *Inter. J. Heat Mass Transfer* **1**, 4, 325–335, Jan. 1961.

The mathematical methods concerning the theory of boundary layer are used to analyze laminar film boiling with variable properties. The boundary-layer equations are transformed to ordinary differential equations, and are solved for compressible flow with variable specific heat. Numerical calculations are performed.

A. Pignedoli, Italy

5009. Ornatsky, A. P., and Kichigin, A. M., An investigation of the dependence of critical thermal loading on weight velocity, underheating and pressure (in Russian), *Teploenergetika* no. 2, 75–79, Feb. 1961.

5010. Alad'ev, I. T., Heat transfer in nucleate boiling (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 233–255.

An analysis is presented of heat transfer in pool boiling. By starting from the Maxwell-Boltzmann distribution law, the Clausius-Clapeyron equation and the force balance on a bubble, author has determined a system of dimensionless parameters which relate the surface superheat to the heat flux. The constants appearing in the correlations were determined empirically.

Experimental data obtained by other investigators are compared with the derived equation and with the predictions of Kruzhilin, Kutateladze, and Tolibinskii. A large number of curves of Δt versus q are given for water, alcohol, *n*-pentane, propane and benzene for pool boiling over a wide range of pressures. A number of similar plots are also shown for boiling of water in copper and chromium steel tubes. The surface superheats predicted by the author's equation agree somewhat better with the experimental data than do the predictions of other Russian engineers. Correlations are also given of the ratio of the wall temperature to the saturation temperature for both pool boiling and for boiling in a tube.

Although 47 references are cited in the paper author does not include extensive work in the field by U.S. investigators. Reviewer believes that the method of analysis employed in the paper sheds little light on the physical phenomenon of boiling, but the method yields a useful correlation for the designer.

R. Viskanta, USA

5011. Zwick, S. A., Growth of vapor bubbles in a rapidly heated liquid, *Physics of Fluids* **3**, 5, 685–692, Sept./Oct. 1960.

The earlier theory of the growth of vapor bubbles in superheated liquids is extended to the situation in which the rate of temperature rise of the liquid is large. Numerical solutions are presented for the early stages of bubble growth for various rates of liquid temperature rise. The asymptotic behavior of a bubble is found ex-

plicity for a temperature rise of the liquid which is linear in time. In this case the bubble radius grows initially as $t^{1/4}$, as in asymptotic solutions found previously for small rates of temperature rise, but then deviates toward a late $t^{3/2}$ variation. (Authors summary).

This report outlines the results of theoretical calculations presented in more detail in cited references to previous work by the author and presents numerical results for bubble growth in water at 1 atm and 18.8 atm for liquid temperature rises of 0.01 °C, 200 °C and 2000 °C per second. R. J. Fritz, USA

5012. Isachenko, V. P., Vzorov, V. V., and Vertogradsky, V. A., Heat transfer at evaporation of water from porous wall in air flow (in Russian), *Teploenergetika* no. 1, 65-72, Jan. 1961.

5013. Konsetov, V. V., Heat transfer during condensation of steam inside horizontal tubes (in Russian), *Inzbener.-Fiz. Zb.* **3**, 6, 9-16, June 1960.

Author considers the formation of a condensed liquid film when steam flows through a horizontal, cooled tube. Account is taken of the entrance region, where the film thickens appreciably, and also of the condensate flowing along the bottom of the tube. Transition from laminar to turbulent flow in the liquid film is also considered. A number of assumptions about friction factors and heat-transfer coefficients are made which strictly apply only to flow through a tube running full. Author obtains solution for the Nusselt number, assuming uniform heat flux along the tube, making certain simplifying assumptions regarding the basic governing equations. Experimental heat-transfer data of the author and others are compared with the theoretical results, giving a scatter one is accustomed to in heat transfer; the experimental results are over-all ones only, and can hardly prove or disprove the theory. No details of the experimental procedure are given.

Y. R. Mayhew, England

5014. Sparrow, E. M., and Siegel, R., Transient film condensation, *ASME Trans.* **81** E (*J. Appl. Mech.*), 1, 120-121, Mar. 1959.

5015. Sparrow, E. M., and Gregg, J. L., A boundary-layer treatment of laminar-film condensation, *ASME Trans.* **81** C (*J. Heat Transfer*), 1, 13-18, Feb. 1959.

5016. Piero, L. S., Zaliznyak, D. V., and Maevskii, E. R., A heat exchanger with movable heat elements (in Russian), *Trudi In-ta Ispol'zovaniya Gaza Akad. Nauk USSR* no. 5, 77-87, 1958; *Ref. Zb. Mekb.* no. 10, 1959, Rev. 12033.

A description is given, and the results obtained, of testing of two heat exchangers with movable heat elements for the heating of the air. The features of this type of heat exchanger are the two chambers, in one of which the preliminary heating of the heat elements takes place by means of the products of the burning gas. On conclusion of this stage the heated elements are utilized continuously by transfer to the other chamber through which the gas to be heated is let through. A small heat exchanger with a capacity of 100 nm³/hour of hot air with a temperature of preheating of up to 700 ° and a pilot-industrial heat exchanger with a capacity of 1500 mm³/hour heated to 500 ° for the air were investigated. China clay balls with a diameter of 12.7 mm were used as movable heat elements. Results are furnished for the tests carried out to study the heat exchange in the layer of movable balls and to determine their resistance. The coefficients of heat emission obtained were several times smaller than those obtained by various investigators when studying heat exchange in a layer of immovable balls, but agree well with Norton's data obtained from a heat exchanger of similar construction. O. S. Vorob'ev

Courtesy Referativnyi Zhurnal, USSR

5017. Kormendy, I., and Jegesi, G., Design problems of heat exchangers for non-Newtonian fluids (in Hungarian), *Gép* **13**, 2, 49-52, Feb. 1961.

5018. Rabinovich, G. D., Heat and mass transfer in a layer of moist material (in Russian), *Inzbener.-Fiz. Zb.* **3**, 12, 29-34, Dec. 1960.

An experimental investigation into the nonsteady process of heat transfer in a layer of moist grain. The paper presents an investigation by standard methods of heat and mass transfer where, as far as the reviewer is aware, most work is done by straightforward empiricism, namely in agricultural engineering. In particular, the paper studies the process of the drying of wheat.

J. Kestin, USA

5019. Tiulpanov, R. S., The effect of pulsations of flow rate on the evaporation of drops of fuel (in Russian), *Inzbener.-Fiz. Zb.* **3**, 5, 119-123, May 1960.

The article considers the question of the effect of velocity pulsations on mass exchange between a stream and particles of fuel. An approximate theoretical solution is obtained for the rate of evaporation of drops of fuel in a gas flow. The accuracy of solution for large drops is completely satisfactory.

From author's summary

5020. Novosel'skaya, N. A., Investigation of the coefficients for the transfer of heat and for the mass of material (in Russian), *Trudi Mosk. In-ta Khim. Mashinostr.* **15**, 99-113, 1958; *Ref. Zb. Mekb.* no. 10, 1959, Rev. 12013.

An attempt is made to determine analytically, on the strength of a single experiment carried out in anisothermal conditions, all the parameters of the transfer of heat and mass applicable to the processes of drying and similar processes which are of importance in the chemical, nutritional and building industries. The coefficient of potential conductivity is determined, the thermo-gradient coefficient and also the mass content, the potential, the mass capacity and the coefficient of the mass conductivity of the absorbed material. Calculation formulas are furnished.

S. M. Il'yashenko

Courtesy Referativnyi Zhurnal, USSR

5021. Mukhlenov, I. P., The kinetics of heat and mass transfer in a foam layer (in Russian), *Zb. Prikl. Khim.* **31**, 9, 1342-1348, 1958; *Ref. Zb. Mekb.* no. 10, 1959, Rev. 12015.

Author investigates the process of heat and mass transfer for a foam layer. In carrying this out the determining criterion, in addition to Reynolds criteria and the geometrical simplexes, should be the specific height of the foam layer. Measurements made of the heat and mass exchanges of a liquid with gases of good solubility in laboratory and industrial plant brought to light the practical absence of influence on the process of the diameter of the apparatus, the diameter of the orifices in the grid and its live section. The coefficients of exchange proved to be approximately linear, increasing with increase in the height of the foam layer.

O. M. Todes

Courtesy Referativnyi Zhurnal, USSR

5022. Weisard, E. L., Pulse method for the measurement of thermal diffusivity of metals, *J. Appl. Phys.* **32**, 1, 40-45, Jan. 1961.

The experiment system consists of a long compound rod formed by two long 3/16-inch o d rods of specimen material with a thin disk-shaped silicon carbide heater, 0.05 inches thick, mounted in between. A current pulse, passed through the rod, causes an initial temperature distribution of $T(x, 0+) = 0$ for $x < -L$ and $x > L$ and $T(x, 0+) = T_0$ for $-L < x < L$ (a square pulse in the region of

the heater). As the region $2L$ becomes small, the initial condition can be considered as an instantaneous plane source. The solution of the one-dimensional, nonsteady diffusion equation with this initial condition and assuming no heat losses or gains is obtained. From this solution, the thermal diffusivity of the metal specimens can be found from the relation $\alpha = C(n)x^2/t_n$, where $C(n)$ is a constant depending on n only and t_n is the time at which the temperature at point x becomes the fraction n of its eventual maximum value. From the temperature-time curves measured at a distance x from the center of the rod, values of t_n are obtained. Substitution into the equation above with the appropriate values of x and $C(n)$ proved experimental values of diffusivity. The experimental runs require less than 60 seconds. Author estimates a maximum error of plus or minus 4%.

Results are presented for four steel specimens at room temperature. The method is easily adapted for measurements at elevated temperatures. A complete description of the experimental apparatus and discussion of the sources of error are included.

R. G. Nevins, USA

5023. Yoshida, I., Thermal conduction in ferroelectric ceramics, *J. Phys. Soc. Japan* **15,** 12, 2211–2219, Dec. 1960.

Apparatuses were constructed and measurements were made on the thermal conductivity of insulating crystals over the temperature ranges from −200° to 600° C. For the lower temperature range, the absolute measurement was used, while for the higher one, the comparative method was adopted. The thermal conductivities of ferroelectric $PbTiO_3$ and anti-ferroelectric $PbZrO_3$ showed showed a rather large step-wise increase, in contrast to the case of $BaTiO_3$, as they passed into the paraelectric state. Discussions are given of the relation between the magnitude of thermal conductivity and the crystal structure. It is pointed out that the anharmonic potential for the smaller ions should be responsible for the scattering of phonons in these substances.

From author's summary by R. L. Young, USA

5024. Minashin, V. E., Subbotin, V. I., Ushakov, P. A., and Sholokhov, A. A., On errors in temperature measurement which are caused by the distortion of isotherms near the thermocouple groove (in Russian), Konvektivnyi i Luchistyi Teploobmen, Moskva, Izdatel'stvo Akad. Nauk SSSR, 1960, 205–220.

A very useful investigation into errors caused by the inevitable presence of grooves or other hollow places near the hot junctions of thermocouples. The main error considered is that due to the distortion of isotherms which cause the thermocouple to measure a somewhat different temperature than that which would exist at the junction if the hollow were not present.

The correction is calculated analytically for some geometries by the assumption that the hollow is perfectly nonconducting compared with the surrounding, conducting metal. Other solutions are obtained by measurements on electrical models (two-dimensional, stainless steel foil) utilizing the analogy between steady-state heat flow and potential distribution of electricity (Laplace's equation).

J. Kestin, USA

5025. Volchok, L. Ya., Thermal inertia of resistance thermometers and thermoanemometers (in Russian), *Izv. Vyssb. Ucbebn. Zavedenii. Energetika* no. 6, 90–99, 1958; *Ref. Zb. Mekb.* no. 10, 1959, Rev. 12023.

A formula is derived for the constant thermal inertia of a resistance thermometer and a thermoanemometer. An experimental determination is carried out on the constant heat inertia of thin wires over a wide range of velocities for the air flow. The values of the constant thermal inertia for a resistance thermometer and a thermocouple are compared and a quantitative evaluation is carried out on the influence of the thickening of the junction of the thermocouple on the magnitude of the thermal inertia.

L. N. Tryakova

Courtesy Referativnyi Zhurnal, USSR

5026. Haas, A., Principles of nonradiometric methods of surface-temperature measurement (in Hungarian), *Meres es Automat.* **8,** 11, 331–337, 1960.

This is an elementary review of some of the heat-transfer mechanisms that vitiate the measurement of surface temperatures by means of thermocouples or resistance thermometers. Equations are presented for estimating the inherent errors in surface-temperature measurements with thermocouples that are (1) led to the surface at right angles and through an isothermal and quiescent medium or (2) embedded under the surface and parallel to it. In both instances the effects of possible temperature gradients in the solid body, perpendicular to its surface, are neglected.

R. Gardon, USA

Combustion

(See also Revs. 4963, 4987, 5024)

5027. Iinuma, K., Spontaneous ignition delay of fuel sprays, Tech. College, Hosei Univ. Tokyo, Rep. 9, 12 pp., Dec. 1960.

This is a carefully planned and executed investigation on the ignition delay of fuel sprays injected into hot air at atmospheric pressure, in the temperature range of 250 to 900° C, for eight hydrocarbon fuels. Author found that the function ignition delay versus temperature, plotted on logarithmic scales, shows a bend in the temperature region between 600 to 700° C, i.e., with increasing temperature the ignition delay decreases first slowly, then rapidly; also the scatter of points decreases. Also, at low temperatures the chemical factors, and at high temperatures the physical factors, have the predominating influence on the ignition delay. Author studied also the influence of the injection pressure and the quantity of fuel injected. He compares his results with those of previous investigators, which are also included in the charts and tabulations.

The experimental arrangement comprises a Bosch single-cylinder, lever-operated injection pump, the lever being actuated by a prestressed helical spring; quantity of injection (about 0.2 cm^3) could be varied by control-rod setting. The conical spray is directed upward into an electric furnace having an internal diameter of about 4 inch, and a length of about 10 inch. Start of injection is detected by an electromagnetic pickup actuated by the feeler pin of the injection nozzle. Start of ignition is detected by a photoelectric tube actuated by the appearance of flame. The elapsed time between the two events was measured first by an electromagnetic oscillograph, but later it was measured directly by an electronic counter circuit comprising a quartz crystal.

Pertinent literature of the last ten years is cited and apparently an effort was made to fill the gaps in the available information, and to coordinate the investigation with other researches on the subject.

K. J. De Juhasz, USA

5028. Simms, D. L., Ignition of cellulosic materials by radiation, *Combustion and Flame* **4,** 4, 293–300, Dec. 1960.

The ignition of cellulosic materials by radiation has been examined. Ingition occurs first in the gas phase; flame is shown to appear well away from the surface. Of the factors affecting ignition, external draughts and the supply of volatiles are likely to determine whether ignition can occur, and the thermal properties, the time at which it will occur; the size of the area irradiated may affect both. The ignition time has been correlated satisfactorily

with the thermal properties over a large range of intensities by assuming that ignition occurs when the temperature of the solid reaches a certain fluid value.

From author's summary

5029. Rumyantsev, S. V., Ermolaev, M. D., Domrachev, V. I., Tikhonova, A. S., and Bulavkin, A. A., Investigation of the cone of flame system of ignition in its application to airplane engines (in Russian), *Trudi Kazansk. Aviats. In-ta* **39**, 112 pp., 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11680.

In the course of a number of years, commencing in 1941, work was carried out in the Kazansk Aviation Institute (K.A.I.) on investigating the cone of flame system of ignition, proposed at that time by co-workers in the Institute of Chemical Physics of the Akad. Nauk SSSR A. N. Voinov and A. S. Sokolik. The essence of the cone of flame system of ignition is contained in the fact that the spark ignites a small part (3–5% by volume) of the charge, which is situated in a special fore-chamber and has a constant (independent of the load) composition for the mixture ($\alpha = 0.8 - 0.9$) which then passes through a special nozzle into the compression chamber and ignites the main part of the charge. The composition of this basic portion of the charge in relation to the load may vary from $\alpha = 0.8$ to $\alpha = 1.6$–2.0. The power source of the ignition enables qualitative control to be effected and at small loads to ignite weak mixtures. This means of control results in considerable fuel economy and has a favorable effect on the capacity of the motor. The work of the K.A.I. deals with: (1) The determination of the optimum parameters of the system of ignition being investigated, (2) the determination of the motor's characteristics, (3) the investigations of the possibilities of the application of this system to engines with direct injection, thus utilizing low-grade fuels, (4) the examination of the working process of the motor combined with a power unit (a motor with forced fuel and a gas turbine). The work was carried out on single cylinder blocks, types of cylinders such as VK-105, PF and ASH-82 being requisitioned. In that part of the work which was devoted to combined units, a cylinder of type V2 was used. The investigation proved fruitful and practically all the problems set by the authors were successfully solved.

G. A. Varshavskii

Courtesy Referativnyi Zhurnal, USSR

5030. Prasad, K. K., Mahadevan, K., and Havemann, H. A., Flame speeds in oscillating gases in a tube, *J. Indian Inst. Sci.* **43**, 1, 26–43, Jan. 1961.

Flow disturbances such as turbulence have long been known to have important effects on flame propagation, affecting in particular the flame speed. Due to the difficulty associated with measuring turbulence and describing its effect on such a complex phenomenon as a flame, it has been pointed out that useful information can be obtained by restricting the study to the interactions of a simple form of flow disturbance and a flame.

This paper describes some experimental studies on the influence of oscillations on an otherwise normal flame propagating from either end of a tube open at one end. The experiments reveal that higher flame speeds invariably result with the imposition of oscillations irrespective of the strength of the combustible mixture. The flame speed increases linearly with frequency of oscillations and the extent of this increase is determined by mixture strength. Rich mixtures show increasing flame speeds with increasing amplitudes, while lean mixtures exhibit a critical amplitude at which maximum flame speed occurs.

From authors' summary

5031. Chen, T.-N., and Toong, T.-Y., Structure and propagation of laminar flames near a heat sink, *Combustion and Flame* **4**, 4, 313–323, Dec. 1960.

A theoretical study is made of a one-dimensional flame with heat loss. Authors assume a first-order reaction following the Arrhenius law, Lewis number unity, and other simplifying assumptions. Both heat loss across the hot boundary in the direction of flow and transverse heat loss from the reaction zone normal to the flow direction are considered. Attention is also given to the effect of varying initial mixture temperature. Equations are derived which may be evaluated in specific cases by iterative procedures. Curves showing sample results are plotted, effects of thermal environment on burning velocity and temperature distribution within flame being shown. Relation to earlier results of Spalding and Mayer is discussed.

R. Friedman, USA

5032. Berlad, A. L., and Yang, C. H., A theory of flame extinction limits, *Combustion and Flame* **4**, 4, 325–333, Dec. 1960.

A theory of nonadiabatic flame propagation, based on one-dimensional, steady conservation relations for energy and for two reactants A and B, is proposed. Analysis differs from usual adiabatic formulation by inclusion of energy loss term, and by a boundary condition limiting the temperature maximum to the adiabatic flame temperature. Arrhenius rate law is approximated by power law. Results obtained for two cases, (1) Lewis numbers of A and B equal zero and (2) Lewis numbers equal to one, yield qualitatively correct behavior of extinction limits. Satisfactory quantitative agreement with experiment is obtained for extinction limits of $C_2 H_2$ —O_2 —N_2 flames.

G. H. Markstein, USA

5033. Shetinkov, E. S., Calculation of flame velocity in turbulent stream, Seventh Symposium (International) on Combustion, London and Oxford, Aug. 28–Sept. 3, 1958; New York, Academic Press, 1959, 583–589.

5034. Shchelkin, K. I., Possible mechanism of amplification of weak shock waves in a zone of turbulent combustion (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk* no. 5, 86–96, 1959.

A schematized combustion chamber of a jet engine is considered and a relationship is established between the intensity of the weak shock wave produced in such a chamber and the increase of the combustion velocity. For this purpose Ia. K. Troshin's results are considered as a starting point [AMR **12**(1959), Rev. 4709 and AMR **13**(1960), Rev. 1501]. It is shown that the combustion velocity depends on the intensity of the shock wave passing through the flame front. It is thus found quantitatively that the propagation velocity of the turbulent combustion increases when a weak shock wave passes through the combustion zone.

The relations deduced permit the determination of the conditions under which the weak shock wave amplifies or attenuates in the turbulent combustion zone. The amplification criterion is equivalent to that of the occurrence of high-frequency vibrations in the schematized combustion chamber.

T. D. Oroveanu, Roumania

5035. Le Grives, E., and Barrere, S., Algebraic methods for the calculation of the composition of equilibrium products high energy reactions (in French), *Rech. Aéro.* no. 68, 31–37, Jan./Feb. 1959.

5036. Cook, E. B., and Smith, R. W., Jr., Computed compositions and thermodynamic properties of deuterium-air flames, U.S. Bur. Mines, Rep. Invest. 5587, 22 pp., 1961.

5037. McClure, F. T., Hart, R. W., and Bird, J. F., Acoustic instability in solid fuel rockets, *ARS J.* **30**, 9, 908–910 (Tech. Notes), Sept. 1960.

5038. Westenberg, A. A., and Favin, S., The theory of a spherical premixed laminar flame, *Combustion and Flame* **4,** 2, 161–171, June 1960.

For a low velocity, premixed laminar flow, only the equations of continuity, diffusion and energy should be considered. These equations in a spherically symmetrical case are given with unimolecular reaction. Numerical solutions obtained on the IBM-650 computer with Lewis number unity are discussed. It is found that the burning velocity increases with decreasing radius of curvature of the flame front. S. I. Pai, USA

5039. Khvostov, V. I., A study of the structure of the process of combustion in a cyclone furnace when the liquid fuel is ignited (in Russian), Issled. Kotel'no-Topochn. Protsessov (MVTU, **94**), 1958, 23–45; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11636.

The results are described of the experimental investigations on the concentrated gas fields inside a cyclone chamber. These results were followed by making photographs of concentration fields in a cyclone chamber working on solar oil. Data were obtained for three variants of the chamber under different regimes of work. The zonal structure of the process of combustion of liquid fuel in a cyclone chamber was established. It was shown that the conditions of working and the role fulfilled by the separate zones in the process of combustion depend largely on the distribution of the air and on the method employed for the feed of the fuel into the chamber. When the liquid fuel is ignited the structure of the process of combustion in a cyclone chamber has the character to an appreciable degree of the flame cone, which depends largely on the vaporability and comparatively easy evaporability of the drops. It was made clear that the principle zone of intensive secondary formation of mixing, in which the mixing of the gases of the central and peripheral zones takes place, is the zone of the outlet nozzle of the chamber, which in the case in question plays the part of an adequately efficient mixer.

Yu. A. Laskhov
Courtesy Referativnyi Zhurnal, USSR

5040. Nagao, F., Kakimoto, H., Matsuoka, Y., and Kato, K., The combustion process in a Lanova air-cell type Diesel engine, *Bull. JSME* **3,** 12, 525–531, Nov. 1960.

High-speed motion picture taken through a glass window in a full-scale engine has led to the following information as to the combustion process in a Lanova diesel engine. The fuel supply into the cell, which depends principally on the fuel injection and the design of the air-cell, has a large influence on the engine performance. The fuel spray, if injected through the injector with a small spray angle toward the mouth of the cell, does not produce much mixture in the main combustion chamber before ignition, which burns slowly until the outflow of hot gases from the cell has improved the mixture formation to such a degree that quicker combustion becomes possible. These processes contribute toward a slow pressure rise in the early stage of combustion and high thermal efficiency. Furthermore, it was ascertained by the high-speed photography that ignition in the main chamber starts generally before that in the cell, and, at the latest, simultaneously with the ignition in the cell. From authors' summary

5041. Nagao, F., Kaikmoto, H., Matsuoka, Y., and Nakatani, H., Photographic study of combustion in a pre-combustion type Diesel engine: Part 1, Combustion in a pre-combustion chamber; Part 2, Combustion in a main combustion chamber, *Bull. JSME* **3,** 12, 507–524, Nov. 1960.

Air motion as well as mixture formation and flame pattern in the pre-combustion chamber of a diesel engine was investigated by means of high-speed photography through a glass window on an engine in operation. By comparing the photographic evidences with the results of test performed under a similar condition, the following conceptions as to the improvement on the combustion were obtained:

The main bulk of fuel should be transferred into the cylinder with the first blast of gas so that combustion in the pre-combustion chamber may take place under lean mixture. This state can be realized by keeping the fuel spray from being wafted back by an incoming air stream and by concentrating a rich mixture near the throat of the chamber. Further, the mixture formation in the chamber is affected to a great extent by an air stream which flows in through the small passage during the compression stroke and then generally creates an air-swirl in the main part of the chamber. An excessively vigorous swirl sometimes results in preventing the earlier outflow of fuel, which brings about much soot formulation.

Using a high-speed motion picture camera, the authors made flame photographs through a glass window set on the cylinder head of a pre-combustion-type diesel engine. For the cases of various directions of gas outflow and also for various shapes of cavity on a piston crown, the process of mixing and combustion of the gas mixture blown out from the precombustion chamber, swirl creation and flame distribution in the cylinder were investigated. In addition, the suitable design of the main combustion chamber for improvement on the combustion was discussed by comparing the results of performance test obtained in a single cylinder engine. From authors' summary

5042. Zelinski, J. J., Matthews, L. J., III, Falk, F., and Bagnall, E. C., The design of combustors for ramjet engines, *Combustion and Flame* **4,** 4, 343–353, Dec. 1960.

A design procedure for ramjet combustors is developed. A model which describes the manner in which the combustor operates is discussed. Experimental programs to support the model are described. The use of the model as a design tool is outlined. Experimental verification of the design procedure is given. From authors' summary

Prime Movers and Propulsion Devices

(See also Revs. 4886, 4970, 5024, 5037, 5042, 5065)

5043. Kalt, S., Thrust termination in solid rocket motors—evaluation of ballistic test data, *ARS J.* **31,** 1, 84–86, Jan. 1961.

Simplified equations are set up for the chamber-pressure transient when thrust-reversing orifices are opened in a solid-propellant motor. Results may be used either to predict pressure-time history for given parameters, or to deduce rate of opening of port area from measured pressure-time curve.

R. Friedman, USA

5044. Ma, C.-C., Advanced uncooled nozzle for solid propellant rocket, *ARS J.* **31,** 3, 368–370 (Tech. Notes), Mar. 1961.

This paper is concerned with the advanced design of an uncooled nozzle for a solid propellant rocket. The results of analytical studies, which include various advanced uncooled nozzle designs based on the requirements of a hypothetical second-stage rocket engine, are presented. However, the nozzle design concept thus developed is applicable to any rocket engine. Among the five possible schemes investigated, the most promising appears to be a design involving a nozzle made of resin-bonded refractory-grade silica or other vitreous materials, the upper part (throat region) inwardly covered with a layer of graphite which in turn is coated with zirconium oxide. Molybdenum containing 0.5 per cent titanium alloy is used as an outer

surface covering for the entire nozzle. Based on a concept of the efficient use of a combination of several materials each of which serves a particular function, the above design for the construction of an uncooled nozzle is believed to be completely novel and results in a nozzle weight which is much lower than that of a conventional fluid-cooled or uncooled nozzle of comparable thrust rating and expansion ratio. From author's summary

5045. Watson, R., Effect of scale size on a rocket engine with suddenly frozen nozzle flow, *ARS J.* **31,** 3, 452–453 (Tech. Notes), Mar. 1961.

Recent analytical work by Bray indicates that single element, chemically reacting systems, e.g., $H_2 \rightleftharpoons 2H$, may suddenly, freeze their composition at some point in a high-speed nozzle and then remain at fixed composition throughout the remainder of the expansion. This sudden freezing or "quenching" phenomenon was also apparent in some theoretical calculations reported by Hall et al, and has been verified experimentally by Wegener. It is the purpose of this note to show qualitatively how Bray's sudden-freezing criterion is related to engine size by the scale factor for geometrically similar engines having nonequilibrium nozzle flows and in which a propellant system is used for which Bray's analysis is valid. From author's summary

5046. Hurrell, H. G., Analysis of injection-velocity effects on rocket motor dynamics and stability, NASA TR R-43, 40 pp., Sept. 1959.

5047. Andrews, E. G. D., and Mottram, A. W. T., Trends in the development of liquid propellant rocket engines, *Aero. Quart.* **10,** 3, 199–210, Aug. 1959.

5048. Holl, R., Lee, R. G., and Boden, J. E., Some tests on an Avon-Canberra installation to measure thrust in flight, Aero. Res. Counc. Lond. Curr. Pap. 518, 23 pp. + plates and figures, 1960.

5049. Short, W., and Sabin, C., A radioisotope propulsion system, *J. Brit. Interplanetary Soc.* **17,** 12, 453–458, Nov./Dec. 1960.

An analysis is made of a proposed low-thrust long-duration space propulsion system. An alpha-emitter (thorium-228) is applied as a coating to a sheet of alpha-absorbing material (beryllium). Thrust results from alpha particles escaping in one direction only. The analysis considers the random direction of emission and self-absorption in the solid. When optimum thicknesses of thorium and beryllium are used, a thrust of 0.0039 dyne/sq cm is attainable, and a maximum acceleration of 2.2×10^{-4} g. The half-life is 1.9 years. Reviewer points out that no consideration is given to the possibility that an alpha-particle originating beneath the surface may knock away a number of atoms, thus causing performance to deteriorate more rapidly.

R. Friedman, USA

Magneto-fluid-dynamics

5050. Gubanov, A. I., and Lun'kin, Yu. P., Couette flow in magnetoplasmodynamics, *Soviet Phys.-Tech. Phys.* **5,** 9, 984–992, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 9, 1053–1060, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

The equations of motion of a viscous plasma in a magnetic field for an arbitrary value of $\omega\tau$ are solved in the simplest case of Couette flow. Cases treated include that of the magnetic field perpendicular to the plates, that of a field parallel to the plates, and also that of a field in an arbitrary direction. A number of features are found in the flow of the plasma that do not follow from magnetohydrodynamics (in which $\omega\tau \ll 1$); for example there is a component of the velocity of the plasma perpendicular to the velocity of the plate; a magnetic field parallel to the velocity decreases the viscous friction, and so on.

From authors' summary

5051. Breus, S. N., Stability of a liquid cylinder carrying current for the case of finite conductivity, *Soviet Phys.-Tech. Phys.* **5,** 9, 960–965, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30,** 9, 1030–1034, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

The stability of a current pinch in a magnetic field is investigated for the case in which no magnetic field is frozen inside the conductor. It is shown that in this case the instability is increased with an increase in the longitudinal magnetic field.

From author's summary

5052. Chong, P., and Inuishi, Y., Conductivity induced by injected electrons in liquid dielectrics, *Technol. Rep. Osaka Univ.* **10,** 414, 545–551, Oct. 1960.

5053. Post, R. F., and Perkins, W. A., Velocity-space plasma instabilities observed in a mirror machine, *Phys. Rev. Letters* **6,** 3, 85–89, Feb. 1961.

5054. Sugata, E., and Kaneda, S., Special type periodic magnetic field for small fluctuation of electron beam, *Technol. Rep. Osaka Univ.* **10,** 419, 601–615, Oct. 1960.

5055. Axford, W. I., The stability of plane current-vortex sheets, *Quart. J. Mech. Appl. Math.* **13,** 3, 314–324, Aug. 1960.

Previous results for the stability of a plane vortex sheet in an incompressible fluid are generalized to include magnetic fields, three-dimensional effects and the presence of walls. Zero viscosity and infinite electrical conductivity are assumed. The current-vortex sheet is given a small sinusoidal disturbance and sufficient conditions then determined for this disturbance to be damped out. Both magnetic fields and the presence of walls are shown to exert a stabilizing effect on the flow.

These results should be of interest to those working on magnetic containment of plasmas. A. L. Loeffler, Jr., USA

5056. Shirokov, M. F., and Prudkovskaya, O. V., Vibrations in electronic-ionic plasmas (in Russian), *Nauchn. Dokladi Vyssb. Shkoly. Fiz.-Matem. Nauk* no. 2, 192–199, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11331.

Authors make their starting point the kinetic equations for the plasma's components. The distribution functions are assumed to be slightly different from the equilibrium functions. Equations are then obtained for the motion of the plasma viewed as a continuous medium, utilizing Maxwell's general equations for transfer. Taken together with the equations for the macroscopic electromagnetic field these equations give a macroscopic description of the plasma. The equations used previously, as the authors show, appear to be less general and the possibility emerges of being able to indicate the limits of their applicability. The equations obtained are of a specialist nature to describe plasma vibrations—electromagnetic waves, acoustic (density waves), magneto-hydrodynamic, electroacoustic, electrodiffusion, and so forth.

G. Z. Gershuni
Courtesy Referativnyi Zhurnal, USSR

5057. Ong, R. S., Characteristic manifolds in three-dimensional unsteady magnetohydrodynamics, *Physics of Fluids* **2,** 3, 247–251, May/June 1959.

5058. Gershman, B. N., and Kovner, M. S., Peculiarities of the quasitransverse propagation of magnetohydrodynamic waves in plasma (in Russian), *Izv. Vyssh. Uchebn. Zavedenie. Radiofizika* **1**, 3, 19–24, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11333.

Contrary to what is the normal procedure in magnetohydrodynamics, Ohm's law is recorded for the full current, containing that portion of the displacement current which is specified by the term with a derivative of the current related to time. The consequence is that the magnitude playing the part of longitudinal conductivity becomes complex. The magnetohydrodynamic wave, for which the indicator of refraction depends on the angle made by the direction of propagating of the wave with the vector of the constant magnetic field, is investigated fully. Analysis shows that the above-mentioned way of taking into consideration the displacement current in the generalized Ohm's law has an appreciable effect on the behavior of a magnetohydrodynamic wave propagating almost perpendicularly to the magnetic field.

Yu. A. Bychkov
Courtesy Referativnyi Zhurnal, USSR

5059. Kaneko, S., Transport coefficients of plasmas in a magnetic field, *J. Phys. Soc. Japan* **15**, 9, 1685–1696, Sept. 1960.

The electric and thermal conductivities, and coefficient of thermal diffusion are computed for plasmas in a magnetic field. Only one species of ion is considered.

Terms of order square root in the mass ratio are kept. The Chapman-Enskog method is used and carried to 6th order. Tables of the coefficients are given. Reviewer does not find that author has made it clear whether large-angle collisions are included, or how the cut off for large impact parameters is made. Thus, he thinks author's claim to 0.1% to 10% accuracy is suspect.

J. Dawson, USA

5060. Gubanov, A. I., and Lun'kin, Yu. P., The equations of magnetoplasmodynamics, *Soviet Phys.-Tech. Phys.* **5**, 9, 977–983, Feb. 1961. (Translation of *Zh. Tekh. Fiz., Akad. Nauk SSSR* **30**, 9, 1046–1052, Sept. 1960 by Amer. Inst. Phys., Inc., New York, N.Y.)

The equations of motion and of the thermal flux are obtained for a plasma in a magnetic field for arbitrary values of $\omega\tau$. The equations are expressed in terms of the components of the hydrodynamic velocity, current, and temperature gradient, both in a special coordinate system in which the magnetic field is directed along one of the axes, and also in an arbitrary rectangular coordinate system.

From authors' summary

5061. Okuda, T., and Yamamoto, K., Characteristics of gas discharge plasma in magnetic field, *Mem. Fac. Engng., Univ. Nagoya* **11**, 1/2, 130–136, Nov. 1959.

The behavior of gas discharge plasma may be affected, in various manners, by the configuration of the magnetic field. In this paper, authors are concerned with its characteristics in longitudinal magnetic field: In uniform field, an appearance of contracted column is characteristic, while in nonuniform field an anomalous plasma distribution in the direction of the magnetic field is distinctive.

From authors' summary

5062. Kanwal, R. P., On magnetohydrodynamic shock waves, *J. Math. Mech.* **9**, 5, 681–695, Sept. 1960.

The jump conditions across a magnetohydrodynamic shock are derived in this paper from a more generalized and rigorous mathematical point of view. Dissipation mechanisms, however, are assumed to be absent.

Among the results obtained beyond those already shown by previous investigator for plane shocks are that the present treatment applies also to curved shocks and that the differential effects of the shock conditions are also discussed. [See De Hoffman, F., & Teller, E., *Phys. Rev.* **80**, p. 692, 1950; Lüst, *Zeit. f. Naturforschung* **8a**, p. 277 (1953); Helfer, H. L., *Astrophys. J.* **117**, p. 177 (1953); Friedrichs, K. O. and Kranzer, H., Notes on magnetohydrodynamics: Nonlinear wave motion, A.E.C. Research and Development Report, NYO-6486 (1958)].

C. T. Chang, Sweden

5063. Lyubimov, G. A., Study of stationary surfaces of jump discontinuities of gas conductivity in an electromagnetic field (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mashinostroenie* no. 5, 9–15, 1959.

The case of shock fronts across which the thermodynamic parameters of current and gas velocity change has been studied quite extensively. In this case, the additional assumption is made that the conductivity of the gas also has a discontinuity at the shock front. This case is studied mathematically for the case where there are electromagnetic fields present.

D. Ter Haar, England

5064. Cole, J. D., Magnetohydrodynamic waves, AFOSR TN 59-13 (California Inst. Technol., Guggenheim Aero. Lab.) 28 pp., Jan. 1959.

Shock waves in an infinitely conducting fluid are studied by means of an idealized piston problem. Switch-on waves are shown to be associated with the discharge of a current sheet. The effect of finite conductivity is studied for both ordinary and switch-on waves. It is shown how current sheets are diffused about the wave front. The effects of nonlinearity are discussed in a qualitative way.

From the author's summary by J. W. Butler, USA

5065. Hurwitz, H., Jr., Kilb, R. W., and Sutton, G. W., Influence of tensor conductivity on current distribution in a MHD generator, *J. Appl. Phys.* **32**, 2, 205–216, Feb. 1961.

Magnetohydrodynamic generators may operate under conditions such that the product of electron cyclotron frequency and mean collision time is not small compared to unity. Accordingly the electrical conductivity is a tensor rather than a scalar quantity. The influence of tensor conductivity on the electrical current distribution has been investigated in two idealized situations, one pertaining to the entrance and exit regions of the generator and the other pertaining to the region near segmented electrodes. The calculations predict modifications of the internal impedance of the generator which can be described in terms of increases in the effective duct length and width.

From authors' summary by J. W. Butler, USA

5066. Reed, T. B., Determination of streaming velocity and the flow of heat and mass in high-current arcs, *J. Appl. Phys.* **31**, 11, 2048–2052, Nov. 1960.

Flow velocities up to 60,000 cm/sec were calculated from temperatures obtained experimentally and from dynamic pressures obtained by subtracting calculated magnetic pressures from total pressures measured relative to ambient pressure in 100- to 300-amp argon and helium arcs. From these calculated flow velocities, total mass-flow rates up to 0.23 gm/sec and enthalpy-flow rates up to 6.7 kw were calculated. Rate of heat transfer due to forced convection, obtained by subtracting calculated heat-transfer rate due to electron condensation from measured total heat-transfer rate, was found to vary as 0.62 power of enthalpy-flow rate, whereas mass-flow rate and enthalpy-flow rate were found to vary respectively as the 1.52 and 2.00 powers of electric current. Results of a simple analog experiment showing flow induced by a divergent electric-current path in mercury are included.

E. L. Knuth, USA

5067. Okada, M., and Maruo, H., Fundamental researches on plasma jet and its application: Part 2, On the characteristics of plasma jet and air plasma cutting, *Technol. Rep., Osaka Univ.* **10,** 417, 579–591, Oct. 1960.

Various fundamental characteristics of plasma jet including electric characteristics of argon-gas-stabilized plasma jet and distribution of gas pressure inside the plasma jet device have been investigated.

On the basis of these results, an air plasma jet device has been developed in which air is to be used as a plasma working gas.

In the application for cutting of industrial materials, many high potentialities of the new air plasma jet have been demonstrated, and therefore it will be expected to be the superior cutting device in the near future. From authors' summary

5068. Kubo, U., and Inuishi, Y., Measurement of gaseous discharge plasma with microwave interferometer: Part 1, *Technol. Rep. Osaka Univ.* **10,** 416–565–577, Oct. 1960.

Aeroelasticity

5069. Johns, D. J., and Parks, P. C., Effect of structural damping on panel flutter—stability of two-dimensional simply-supported panels using linear piston theory, *Aircr. Engng.* **32,** 380, 304–308, Oct. 1960.

Authors show that structural damping may be destabilizing for title problem. Reviewer believes that this effect is not likely to be important for practical configurations, where the effective structural damping coefficient is generally that of the material (typical value: $g = 0.005$). Paper contains extensive bibliography. J. W. Miles, USA

5070. Molyneux, W. G., Wind tunnel flutter tests on an M-planform wing, Aero. Res. Counc. Lond. Curr. Pap. 509, 7 pp. + figs., 1960.

Tests were made in a low-speed open jet wind tunnel to obtain the effects on the flutter of an M-wing of variation of inertia parameters of a nacelle at the wing kink. The effect of fuselage mobility was also investigated.

The results show that for the particular wing tested a nacelle mass up to about 0.8 of that of the wing can be tolerated at the kink without a significant adverse effect on flutter, and, in general, an aft position of nacelle c.g. is to be preferred. Lower flutter speeds are associated with symmetric fuselage freedoms than with antisymmetric ones.

The latter result is thought to apply to M-wings in general but the majority of the results obtained cannot be generalized. From author's summary

5071. Broglio, L., Buongiorno, C., Ponzi, U., and Ravelli, G., Experimental investigation on delta wing and complete configuration models for aeroelastic similarity problems at supersonic blow-down wind-tunnel, AFOSR TN 59-954 (Universita di Roma, Scu. Ingegneria Aero. SIAR 50, TN 7), Mar. 1959.

The general remarks made in AMR **14**(1961), Rev. 3287 apply to this review. Paper treats aeroelastic problems on triangular wing models at $M = 3.01$ and on a complete airplane configuration at $M = 3.12$.

It is shown that the chordwise deflection of a triangular wing has very minor importance on the aerodynamic characteristics. Main parameter governing the aerodynamic coefficients is the ratio of stagnation pressure to the modulus of elasticity.

A preliminary design for a magnetic support for dynamic stability tests is presented. By balancing the elastic reactions of the support by magnetic forces, model behaves like a free model. H. J. Ramm, USA

5072. Rodden, W. P., On vibration and flutter analysis with free-free boundary conditions, *J. Aerospace Sci.* **28,** 1, 65–66 (Readers' Forum), Jan. 1961.

5073. Wood, C. J., Transonic buffeting on aerofoils, *J. Roy. Aero. Soc.* **64,** 599, 683–686, Nov. 1960.

Aeronautics

(See also Revs. 4859, 4895)

5074. Christopher, P. A. T., The stability of the short-period motion of an airframe having non-linear normal force and pitching moment curves, *Aero. Quart.* **11,** 3, 255–268, Aug. 1960.

The analysis of the stability of second-order nonlinear systems by Poincaré's method of singular points in the phase plane is briefly presented and used to determine stability criteria for the short-period motion of an airframe having nonlinear normal force and pitching moment curves. These results are then compared with those obtained by the application of quasi-linear stability theory. From author's summary by Y.-Y. Yu, USA

5075. Staufenbiel, R., Computation of the shortest landing distance (in German), Dtsch. Versuchsanstalt Luftfahrt, Ber. 130, 72 pp., July 1960.

Formulas for the various phases of an aircraft landing maneuver are given. The landing distance is computed in terms of the approach speed and path angle and the aircraft and engine parameters. Conventional landing technique is assumed and details of aircraft handling to minimize landing distance are discussed.

Reviewer also recommends NASA TN D-323, which discusses a related problem (power-off landing technique) by similar techniques and presents flight data for nonconventional landings. There is also an excellent discussion of the subject problem in a report by M. Arens, "An analysis of the landing maneuver of winged aircraft," *Bull. Res. Council Israel* **9C,** 1–10, Feb. 1961. W. Daskin, USA

5076. Solonin, S. V., Navigational problems connected with jet flows (in Russian), *Trudi Leningr. Gidrometeorol. In-ta* no. 8, 51–55, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12068.

The question is investigated of taking into account the wind in the upper troposphere and the lower stratosphere for the purpose of determining the most favorable (in point of time) route for passenger-carrying jet airplanes. Author starts from the assumption that in the free atmosphere the wind can be taken to be geostrophic everywhere, including the zones of jet flows. The flight of the airplane is examined as taking place in a horizontal plane with a constant air velocity. Author obtains an equation linking the change with time of the course of travel of the airplane with the altitude of the isobaric surface when in flight on the minimum time trajectory. This equation together with the equations for the motion of the airplane determine the trajectory being sought. Author considers that, taking into account the speeds of modern passenger planes, it is possible to compile charts for the prognoses of the flight for some mean moment of time and to investigate the baric field as being stationary along the whole of the flight's route. V. V. Bykov

Courtesy Referativnyi Zhurnal, USSR

5077. Smiley, R. F., and Horne, W. B., Mechanical properties of pneumatic tires with special reference to modern aircraft tires, NASA TR R-64, 58 pp., 1960.

See AMR **11**(1958), Rev. 2140.

5078. Esgar, J. B., and Morgan, W. C., Analytical study of soft landings on gas-filled bags, NASA TR R-75, 30 pp., 1960.

See AMR **14**(1961), Rev. 1891.

5079. Chaplin, H. R., Ground effect machine research and development in the United States, David W. Taylor Mod. Basin Rep. 944, 32 pp., Dec. 1960.

Ground effect machine (GEM) investigations in the United States have grown at an astonishing rate during the past three years. At present they involve some forty commercial firms, laboratories, etc., and represent an increasingly significant investment of capital and engineering manpower. These investigations are thus far largely exploratory in nature. They represent a considerable variety of approaches, both as to types of ground cushion phenomena employed, and vehicle applications envisioned. The various ground cushion concepts are reviewed, in terms of elementary principles of fluid mechanics. All of the concepts are shown to give direct relationships between vehicle performance and vehicle size/height ratio. GEM developments are therefore expected to tend ultimately toward large ocean-going vehicles operating very close to the surface. Some of the outstanding advantages and problems of the various ground cushion concepts are discussed, but present knowledge does not support strong opinions as to which of the concepts will prevail.

From author's summary

5080. Hirsch, A. E., The hovering performance of a two-dimensional ground-effect machine over water, David W. Taylor Mod. Basin Rep. 1426, 17 pp., Oct. 1960.

The performance in hovering of the Hydroskimmer, a rectangular ground-effect machine with longitudinal side curtains, designed for overwater operation, has been investigated. Indications are that the simple momentum theory for thin jets adequately describes the performance in hovering of this craft over both a fluid and solid surface. The performance of the Hydroskimmer is compared with that of other hovering craft.

From author's summary

Astronautics

5081. Faulkner, F. D., Complete elementary solution to some optimum trajectory problems, *ARS J.* **31**, 1, 33–39, Jan. 1961.

An elementary method is given for solving problems in rocketry, involving the optimum relation for fuel consumption, time and position. Drag negligible, gravity constant, are the simplifying assumptions. The method can be applied to initial design problems for an estimate of thrust and fuel requirements for a certain specific task, such as to gain a given altitude or to intercept a target.

From author's summary by J. Mandelker, USA

5082. Warden, R. V., Ballistic re-entries with a varying $W/C_D A$, *ARS J.* **31**, 2, 208–213, Feb. 1961.

An analytical method is presented for evaluating the velocity and deceleration of a ballistic vehicle, with a variable weight-drag ratio, produced by the process of either ablation or fluid injection into the boundary layer. Drag-area and -coefficient also vary with altitude. Gravity and centrifugal forces are neglected; this allows a closed form solution of the re-entry trajectory.

From author's summary by J. Mandelker, USA

5083. Gorelov, Iu. A., On two classes of plane extremal motions of a rocket in vacuum, *Appl. Math. Mech.* (*Prikl. Mat. Mekh.*) **24**, 2, 434–442, 1960. (Pergamon Press, 122 E. 55th St., New York 22, N.Y.)

A study of the motion of rockets along curvilinear trajectories which are extremal in time and expenditure of mass is presented. In this study the variational equations for the given motion are set up and solved. Motion for an active segment of a trajectory, i.e., one for which the mass of the rocket may vary with time, is considered. Also, the solution of the problem is sought for plane motions in horizontal and vertical planes separately (assuming absence of the influence of aerodynamic forces).

The paper is largely mathematical in nature, and a prior familiarity with the principles of variational calculus is recommended. A suggested reference on variational calculus is Courant and Hilbert "Methods of mathematical physics," Vol. I.

I. J. Eberstein, USA

5084. Lass, H., and Solloway, C. B., Motions of a satellite of the Moon, *ARS J.* **31**, 2, 220–222, Feb. 1961.

Perturbation of the motion of a close lunar satellite is shown to depend primarily on the Moon's oblateness, the Moon's longitudinal asymmetry and the Earth. Using the appropriate potential, authors derive equations for variation of the elements using the averaging method of Kryloff and Bogolyubov. Results are interpreted to show that a close polar satellite remains essentially polar over a long interval—and the orbit may be considered stable. An integral of the motion is obtained such that accurate observations of a nearly circular orbit (not necessarily polar) will yield values of two fundamental constants related to the principal moments of inertia of the Moon.

J. Lorell, USA

5085. Hohmann, W., The attainability of heavenly bodies, NASA TT F-44, 104 pp., Nov. 1960.

5086. Musen, P., The theory of artificial satellites in terms of the orbital true longitude, *J. Geophys. Res.* **66**, 2, 403–409, Feb. 1961.

In a previous article [*J. Geophys. Res.* **64**, 2271–2279, 1959] author explains Hansen's theory applied to the motion of an artificial satellite of the Earth, as it has been used by NASA in much of the latter's determination work. Present article extends and improves the earlier theory. Instead of the disturbed eccentric anomaly, the present development is in terms of the orbital true longitude.

J. Lorell, USA

5087. Koikin, J. A., The influence of tidal friction on the motion of an Earth's satellite (in Russian), *Astron. Zh.* **35**, 2, 297–300, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11158.

An evaluation is made from below on the basis of the statistical theory of tides for the time taken for an artificial satellite to drop out of orbit under the action of tidal friction

$$T > \frac{2}{L}\left[\left(\frac{D_0}{\alpha}\right)^{4.5} - 1\right]$$

where T is the time of drop of the satellite, D_0 the initial distance between the centers of the masses of the Earth and the satellite, α the equatorial radius of the Earth, $L = 1.4 \times 10^{-32}$ m sec^{-1}, m the satellite's mass. In making this evaluation it was assumed that the satellite's orbit is circular, that the Earth is spherical, absolutely solid and covered by ocean of identical depth over the whole of its surface. The diminution of the satellite's energy is dependent on the resistance to the motion of the tidal wave relative to the fundamental mass of the liquid. A numerical example indicates that the influence of the effect being investigated on the duration of the existence of the artificial satellite is inappreciably small.

Note by the abstractor: Under the letter E in formula (20) it is apparently necessary to imply the full and not the kinetic energy of the satellite. The deduction regarding the swell influence of the tidal friction holds good nevertheless, but constant L in the evaluation formula should be doubled.

V. A. Sarychev

Courtesy Referativnyi Zhurnal, USSR

5088. Hoelker, R. F., and Silber, R., Injection schemes for obtaining a twenty-four hour orbit, *Aerospace Engng.* **20,** 1, 28–29, 76, Jan. 1961.

5089. Baker, R. M. L., Jr., Recent advances in astro dynamics—1960, *ARS J.* **30,** 12, 1127–1140, Dec. 1960.

5090. Riddell, W. C., Launch parameters for interplanetary flights, *ARS J.* **30,** 12, 1170–1171 (Tech. Notes), Dec. 1960.

5091. Sommer, S. C., and Tobak, M., Study of the oscillatory motion of manned vehicles entering the earth's atmosphere, NASA Memo 3-2-59A, 30 pp., Apr. 1959.

Ballistics, Explosions

(See Revs. 4871, 4880, 5082)

Acoustics

(See also Revs. 4717, 4722, 4723, 4806, 4923, 4924, 4927, 4962, 4966, 5037, 5130)

5092. Leslie, C. B., and Sorensen, N. R., Integral solution of the shallow water sound field, *J. Acoust. Soc. Amer.* **33,** 3, 323–329, Mar. 1961.

The integral expression for the sound field due to a point source located in a shallow water layer overlying a semi-infinite, liquid-type bottom is transformed into the complex plane so that evaluation by use of a high-speed digital computer becomes possible. The important sector of horizontal distances between zero and several times the water depth is considered. One typical situation studied in detail up to a frequency of 500 cy/sec involves a source placed at a depth of 50 ft in water 60 ft deep above a hard sand bottom. More details on the use of the computer and the results obtained are available in Naval Ordnance Laboratory Report no. 6656 by N. R. Sorensen.

R. Heller, USA

5093. Kazantsev, V. F., The motion of gaseous bubbles in a liquid under the influence of Bjerknes forces arising in an acoustic field, *Soviet Phys.-Doklady* **4,** 6, 1250–1254, May/June 1960. (Translation of *Dokladi Akad. Nauk SSSR (N. S.)* **129,** 1, 64–67, Nov./Dec. 1959 by Amer. Inst. Phys., Inc., New York, N.Y.)

Bjerknes forces are the mean interaction forces between immersed bodies whose volumes are pulsating in time. Author computes these forces for gas bubbles in a liquid where the bubble volume changes occur in response to an imposed sound field. Some experiments by the author in which the motions of bubbles in a known sound field are directly measured are in good agreement with the calculation, showing that Bjerknes forces are important in the ultrasonic bubble coalescence process.

A. Kistler, USA

5094. Phillips, O. M., On the generation of sound by supersonic turbulent shear layers, *J. Fluid Mech.* **9,** 1, 1–28, Sept. 1960.

Author develops a theory for the generation of sound from a turbulent shear flow in the limit of Mach numbers approaching infinity. Only the radiation component of the pressure is considered and the predominant sound-generating mechanism emerging from the analysis is radiation by eddies moving at supersonic speeds with respect to the free stream, so-called eddy Mach waves. Radiation of a particular wavelength and frequency originates primarily from regions in the shear layer where the local mean velocity is equal to the wave phase velocity in the stream direction. Author finds that $\langle (p - \bar{p})^2 \rangle$ varies as M^{5} in the limit of M approaching infinity, but this result is most probably sensitive to assumptions as to how the properties of the shear layer vary with Mach number. A strong directionality of the radiated sound field is discovered. The predictions of the theory are qualitatively supported by some existing experimental results.

Reviewer feels this paper is a major contribution to our understanding of the aerodynamic noise problem.

A. Kistler, USA

5095. Powell, A., Concerning the noise of turbulent jets, *J. Acoust. Soc. Amer.* **32,** 12, 1609–1612, Dec. 1960.

The suggestion that the noise generators of turbulent jets undergo convection effects which are limited in such a way as to follow a similarity behavior leads directly to a resolution of Lighthill's paradox, namely, the problem of accounting for the noise power depending upon the eighth power of the jet velocity simultaneously with the gross directional bias. This hypothesis is shown to be at least plausible to a first approximation owing to the general velocity field of the jet having typical dimensions comparable to a fraction of a wavelength; an important corollary is the expectation of appreciable refraction effects. Aspects relevant to the directional peaks of the higher frequencies being less pronounced and located further from the jet axis, and of the slow frequency rise, are briefly discussed.

From author's summary

5096. Lippert, W. K. R., Method of determining the propagation parameters of wave motion in acoustical material (in English), *Acustica* **9,** 6, 435–440, 1959.

5097. Powell, A., Propagation of a pressure pulse in a compressible flow—coda, *J. Acoust. Soc. Amer.* **32,** 9, p. 1116, Sept. 1960.

5098. Rivin, A. N., Systematic error in reproducing unit sound pressure, *Measurement Techniques* no. 5, 582–585, Feb. 1960. (Translation of *Izmeritel'naia Tekhnika, USSR* no. 5, 71–74, Sept./Oct. 1958 by Instrument Society of America, Pittsburgh 22, Pa.)

5099. Mouric, K., Present state of the art of noise damping in simple walls (in Czech.), *Stavebnicky Casopis* **8,** 5/6, 363–374, 1960.

5100. Tamm, K., and Weis, O., Experiments with periodic waves, exponential and complex sound fields in a bounded solid (in German), *Akustische Beihefte* no. 1, 275–288, 1959.

5101. Ryll-Nardzewski, J., Methods of measuring the loudness of compound acoustic signals, Proc. Vibration Problems no. 2, 3–16, 1959.

5102. Eden, D. D., Lindsay, R. B., and Zink, H., Acoustic attenuation and relaxation phenomena in steam at high temperature and pressure, *ASME Trans.* **83** A (*J. Engng. Power*), 1, 137–144, Jan. 1961.

The purpose of this investigation is the measurement of ultrasonic attenuation in steam at high temperatures and at pressures above atmospheric, in the search for possible relaxation mechanisms. An acoustic fixed-path interferometer has been constructed to allow attenuation measurements up to temperatures of the order of 450°C and pressures around 100 atmospheres. Relaxation peaks have been identified in the pressure range from 1 to 10 atmospheres and over the temperature range from 235 to 400°C. These correspond to relaxation times of the order of 2×10^{-5} sec, which is probably too small to be of significance in the high-

speed flow properties of steam under extreme conditions. The results reported here are in order-of-magnitude agreement with those obtained by Huber and Kantrowitz using the impact tube method. From authors' summary

5103. Snowdon, J. D., Steady-state behavior of the dynamic absorber, *J. Acoust. Soc. Amer.* **31,** 8, 1096–1103, Aug. 1959.

5104. Holman, J. P., and Mott-Smith, T. P., The effects of constant-pressure sound fields on free-convection heat transfer from a horizontal cylinder, *J. Aero/space Sci.* **26,** 3, 188–189 (Readers' Forum), Mar. 1959.

5105. McSkimin, H. J., Pulse superposition method for measuring ultrasonic wave velocities in solids, *J. Acoust. Soc. Amer.* **33,** 1, 12–16, Jan. 1961.

The frequently used pulse method of measuring ultrasonic wave velocities in solids involves a high-frequency quartz crystal transducer cemented to one end of a specimen having parallel end faces. The phase shift for waves reflected from the transducer must be considered for highest accuracy. It is shown that combining several measurements of phase delay (at two frequencies differing by approximately 10%) with a theoretical analysis of the reflection phase angle makes possible a determination of velocity to within one part in 5000 for round-trip delays greater than 5 μsec. Indirectly, the approximate thickness of the cement bond between transducer and specimen can be determined. The advantages of the method for making measurements as a function of temperature or pressure are discussed. From Author's summary

5106. Gucer, D. E., Design for the application of ultrasonic energy to metallurgical processes (in English), *Bul. Istanbul Tekn. Univ.* **13,** 2, 69–82, 1960.

This paper describes an experimental set-up for measuring and controlling the vibrational energy input into a liquid-solid mixture inside a high-temperature hydrogen furnace. The design is applicable to solidification of metal melts, to dispersion of solid particles in liquids, as well as to liquid sintering. Calculations are made for a WC-Co liquid sintering model.

The control of input energy is achieved by proper acoustic design of the wave path and the measurement of dynamic strain amplitudes along this path. A method of calculation of the energy transmitted through the interface between the transmission bar and liquid sintering specimen and of the dynamic variables inside the specimen is proposed. From author's summary

5107. Higgs, R. W., and Litovitz, T. A., Ultrasonic absorption and velocity in molten salts, *J. Acoust. Soc. Amer.* **32,** 9, 1108–1115, Sept. 1960.

5108. Cherry, L. B., Electro-magnetic induction damping of vibratory motion, *Noise Control* **6,** 5, 8–11, Sept./Oct. 1960.

Micromeritics

(See also Rev. 5019)

5109. Friedlander, S. K., Similarity considerations for the particle-size spectrum of a coagulating, sedimenting aerosol, *J. Meteorol.* **17,** 5, 479–483, Oct. 1960.

This is a basic attempt to predict the drop-size distribution for the following model: drops smaller than those of interest coalesce steadily to feed material into the small-size end of the "spectral" range; spectral transfer toward the large-size end occurs by pair coalesence. Material is lost at the large-size end by gravitational fallout. Dimensional reasoning is applied in somewhat the same way as in turbulence spectral theory.

Reviewer finds the final results qualitatively plausible, but is unable to follow the dimensional arguments.

S. Corrsin, USA

5110. Hewitt, D. M., Richardson, J. F., and Gliddon, B. J., Hydraulic conveying of solids in vertical pipes, *Trans. Inst. Chem. Engrs.* **39,** 2, 93–100, Apr. 1961.

Data are presented for the vertical transport of a range of materials of varying density and size distribution, the other variables being pipe diameter, slurry concentration, and velocity.

From authors' summary

5111. Crosse, P. A. E., Lucas, D. H., and Snowsill, W. L., Instrument for recording the dust nuisance emitted by chimneys, *J. Sci. Instrum.* **38,** 1, 12–16, Jan. 1961.

A new type of instrument has been designed to measure and record the likelihood of dust emitted from a chimney causing nuisance at ground level. The instrument measures the difference between the optical obscuration produced by a clean glass surface and that produced by the same glass surface when dust collected from a flue duct has settled on it. The method of collection is based on the assumption that vertical surfaces are more important in pollution than horizontal surfaces, and that coarse particles are more likely to cause nuisance than fine particles. Care has been taken in the design to eliminate errors normally associated with optical instruments of this nature, for example those caused by a dirty optical system, ageing photocells and lamps, and main voltage and temperature fluctuations.

The instrument has a novel recording method which enables it to operate over an unusually large range of signal levels. It will facilitate the control of dust emission from industrial chimneys.

From authors' summary

5112. McDonald, J. E., An aid to computation of terminal fall velocities of spheres, *J. Meteorol.* **17,** 4, 463–465, Aug. 1960.

5113. Nelson, P. A., and Stevens, W. F., Size distribution of droplets from centrifugal spray nozzles, *AIChE J.* **7,** 1, 80–86, Mar. 1961.

This paper discusses methods of measuring, expressing, and correlating drop-size data which the authors found useful in their study of grooved-core centrifugal spray nozzles. They emphasize that the distribution functions are not derived from natural laws, but are made to fit experimentally obtained data. They investigate several types of normal distributions, namely (1) log-normal, (2) square-root normal, and (3) upper-limit distribution; the first two are two-parameter, the third a three-parameter function. In present investigation authors found the square-root-normal distribution best adapted to represent the actual data. The experiments were executed by freezing the spray droplets in liquid nitrogen on their emergence from the nozzle and subsequently sieving them through nine sieves. The advantage is pointed out that all of the spray is collected and sieved, thus avoiding sampling errors; microscopic examination did not reveal any fracturing of the frozen droplets. This technique is restricted to liquids having melting point above $-20\,^{\circ}C$, and to sprays with volume median drop diameter greater than 38 micron. In 114 runs, a wide range of nozzle sizes, spraying pressures, and liquid properties were covered; altogether seven liquid hydrocarbons were used, covering a range of viscosities of 10:1, densities 4:1, and surface tension 3:1.

The test results were correlated by dimensional analysis, the nondimensional quantities being formed of combinations of: air-core diameter, nozzle orifice diameter, mass-average velocity (axial, and tangential), orifice roughness factor, density, viscos-

ity, and surface tension. A number of charts are shown, each representing a large number of test data, which cluster around median curves in close proximity, attesting the validity of assumptions. It is to be noted that the properties of the gas medium were not taken into account.

This is a carefully executed research project, representing a contribution of value to existing literature on sprays and nozzles.

K. J. De Juhasz, USA

5114. Morrell, G., Critical conditions for drop and jet shattering, NASA TN D-677, 13 pp., Feb. 1961.

When a liquid drop or jet is subjected to a gas stream exceeding some critical value of velocity, it will disintegrate and break up. This behavior has important effect on combustion, e.g., in rocket engines, where it may cause sudden change in combustion intensity and amplification of pressure waves. A knowledge of the laws governing drop and jet shattering should help in developing a theory of nonlinear oscillatory combustion. Author reviews recent work in this field, by Hanson, Domich,and Adams, Rabin and Lawhead, Hinze, Gordon, and others.

Based on the philosophies and results of previous researches, the author presents a generalized model describing the threshold conditions for breakup of jets and drops subjected to a step change in velocity with variable flow duration. For flow durations that are long compared with the natural period of the liquid system, a deformation mechanism is postulated; for flow durations that are short compared with the natural period, a stripping mechanism governed by the tensile properties of the liquid is assumed. The breakup conditions predicted by the theory for an assumed critical displacement are in general agreement with published experimental data.

The main body of the paper is largely theoretical and mathematical, but not too difficult to follow. This paper provides a useful background and framework for the study of the voluminous literature on the subject, which deals with specific aspects, and definite applications, of jet and drop breakup.

K. J. DeJuhasz, USA

5115. Welander, P., A theoretical power-law for the size distribution of small particles or drops falling through the atmosphere, *Tellus* **11,** 2, 197-201, May 1959.

5116. Larras, J., Experimental research on scour at the base of a bridge pier (in French), *C. R. Acad. Sci. Paris* **251,** 3, 330-331, July 1960.

Porous Media

(See also Revs. 4969, 5012)

5117. Haushild, W., and Kruse, G., Unsteady flow of ground water into a surface reservoir, *Proc. Amer. Soc. Civ. Engrs.* **87,** HY 1 (*J. Hydr. Div.*), 13-20, Jan. 1961.

Prediction of the water table position and the amount of water discharged where ground water flows into a body of surface water has not been exact. The nonlinear differential equation is difficult to solve. Paper presents approximate solutions of this problem by two different approximation methods. First, the differential equation is linearized and a solution is obtained. Then, the nonlinear terms are computed with the help of the solution of the linear equation and a further approximation is obtained.

The physical boundary conditions employed imply that the water table meets the edge of the body of surface water. No surface of seepage is allowed for.

From authors' summary by A. E. Scheidegger, Canada

5118. Lyu, T.-T., Nonstationary motion of ground water in nonhomogeneous water-bearing strata (in Russian), *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, Mekh. i Mash.* no. 6, 127-129, Nov./Dec. 1960.

Author considers the problem of the motion of a water table along a straight base-line (coordinate x) if the permeability changes abruptly from one constant value to another at $x = l$. At $x = 0$, there is contact with an open reservoir (water height H_0). The solution is obtained by the use of Laplace transforms.

A. E. Scheidegger, Canada

5119. Leczfalvy, S., Estimating the curve of depletion and the time of exhaustion of artesian wells (in Hungarian), *Hidrológiai Közlöny* **40,** 3, 227-233, June 1960.

Author differentiates between artesian wells discharging under hydrostatic, geostatic and gas pressure. These forces are generally combined in nature. In this paper hydrostatic pressures only are considered. Author derived the rate of exhaustion of an aquifer with and without recharge, considering one and more discharging wells. Equations for the estimated time-depletion curve, time of exhaustion and half period of exhaustion (at recharge) are given.

A. L. Simon, USA

5120. Lukomskaya, M. A., The solution of a system of equations describing the steady-state, plane movement of ground water where the filtration law is nonlinear (in Russian), *Inzhener.-Fiz. Zh.* **3,** 148-151, Mar. 1960.

Author considers the following law of filtration

$$\operatorname{grad} H = -\Phi(w)\,\vec{w}/|w|$$

where $\vec{w}$ is the filtration velocity vector, H the hydraulic head and Φ a differentiable function. She shows that the equation for the flow velocity becomes simple if a suitable transformation of the variables is made.

A. E. Scheidegger, Canada

5121. Kim, V. Yu., On a unique integral method in the theory of nonstationary filtration of fluids in porous media (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **30,** 126-130, 1960.

A special case of nonsteady-state flow through porous media is investigated. It is assumed that the diffusivity equation describing such flow has the following form:

$$a^2 \frac{1}{x^s} \frac{\partial}{\partial x}\left(x^s \frac{\partial p}{\partial x}\right) = \frac{\partial p}{\partial t}; \qquad S = 0,\ 1,\ 2.$$

The solution is effected by transforming the above equation into an integral equation and solving the latter.

A. E. Scheidegger, Canada

5122. Guseinov, G. P., and Nasrullaev, I. A., The motion of an elastic liquid in a stratum whose width is variable (in Russian), *Izv. Vyssh. Uchebn. Zavedenii. Neft'i Gaz.* no. 9, 61-65, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12126.

A solution for the equation covering the case of uniform filtration

$$\kappa \frac{\partial}{\partial x}\left(b \frac{\partial p}{\partial x}\right) = b \frac{\partial p}{\partial t} \qquad [1]$$

which determines the function of pressure p when there is an elastic regime in the stratum with a smoothly changing width b. Here κ is the coefficient of piezoconductivity, t is the time. Equation [1] is solved by means of the special assumption that $b = y^n$ $(y = ax + b)$ and with boundary and initial conditions of

$$p = f_1(\tau) \quad \text{with} \quad y = y_1$$
$$p = f_2(\tau) \quad \text{with} \quad y = y_2 \ (\tau = \kappa a^2 t)$$
$$p = g(y) \quad \text{with} \quad t = 0$$

For the solution of the given boundary problem use is made of the Fourier method and of Duhamel's theorem. Authors claim that the case $b = \exp bx$ is also solvable in analogous fashion. A numerical example with $b = ax + b$ is examined on the basis of the formulas derived from the study. For this numerical example a graph is drawn to show the changes in yield in the gallery situated on the line $x = 0$. For purposes of comparison a graph is also drawn to show the changes in yield of the gallery calculated by the known formula for a constant width, equal to the mean value of the width with the other conditions unchanged.

G. S. Salekhov
Courtesy Referativnyi Zhurnal, USSR

5123. Kosheleva, I. M., and Kusakov, M. M., Displacement from a porous medium of mutually soluble hydrocarbon liquids (in Russian), *Trudi Mosk. Neft. In-ta* no. 22, 158–169, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12131.

Results are given of the investigations of the process of the mutual displacement of nonpolar liquids and of the investigation on the displacement of petroleum by a nonpolar liquid. The conclusion reached from the experiments was that in the case of the displacement of one nonpolar liquid by another, provided there are no signs of meniscus forming on their division boundary, the velocity of the displacement and the yield depend on the geometry of the porous area of the porous medium and on the viscosities of the liquids. When petroleum is displaced by a nonpolar liquid the displacement process depends not only on the geometry of the porous area and the viscosities of the displacing liquid and the liquid being displaced but also on the adsorption processes, functioning on the division boundary: petroleum–solid phase.

M. V. Filinov
Courtesy Referativnyi Zhurnal, USSR

5124. Kusakov, M. M., and Kosheleva, I. M., The influence of capillary forces on the process of the displacing of a hydrocarbon liquid by water from a hydrophilic porous medium (in Russian), *Trudi Mosk. Neft. In-ta* no. 22, 170–180, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12132.

Experimental data are published, obtained in the investigation of the displacement process of a nonpolar hydrocarbon liquid (substitute for petroleum) by distilled water (substitute for water in the stratum) from artificial core material with a porosity of 25 to 30% and a permeability of 200 to 750 millid'Arcy. The experimental results and their comparison are presented in the form of graphs and tables. The authors conclude, on the basis of analysis of the results, that the petroleum yield from a hydrophilic porous medium is governed fundamentally by the action of capillary forces and does not depend on the displacement pressure, the permeability of the medium or on the viscosity of the moving liquids. The factors mentioned only exercise influence on the velocity of the displacement process.

M. V. Filinov
Courtesy Referativnyi Zhurnal, USSR

5125. Kusakov, M. M., and Kosheleva, I. M., Displacement of petroleum by water from a hydrophobic porous medium (in Russian), *Trudi Mosk. Neft. In-ta* no. 22, 181–197, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12133.

A description is given of the procedure used and of the results from the experimental investigation of the displacement process of "tuimazinsky" petroleum by distilled water, and water from the stratum, from artificial core material and from natural cores with and without a combined water content. In the experiments the artificial cores used had a permeability of from 100 to 1000 millid'-Arcy and a porosity of 25 to 32%, while the natural cores had a permeability of from 100 to 1700 millid' Arcy and a porosity of 18 to 25%. The dimensions of the cores: length of the order of 3 cm, diameter 1.4cm. An analysis of the investigations carried out made it possible to establish that when petroleum is displaced by water from a hydrophobic porous medium an optimum drop in pressure takes place at which the petroleum yield reaches its maximum.

M. V. Filinov
Courtesy Referativnyi Zhurnal, USSR

5126. Kusakov, M. M., and Gimatudinov, Sh. K., Capillary displacement of petroleum by water from natural cores (in Russian), *Trudi Mosk. Neft. In-ta* no. 22, 198–206, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12134.

The objects of the enquiry were the study of the nature of the surface of the sand strata saturated with petroleum and water, the distribution of the petroleum and water in the porous mass and also an investigation of the displacing capacity of strata waters of different compositions. Sand deposits, petroleum and strata waters from the Zol'ni oil field were used for the investigation. Results of the experiments were presented in the form of graphs. The conclusion to be drawn from the experiments on the capillary displacement of petroleum by water is that the velocity of displacement of the petroleum by the water depends on the quality of the water doing the displacement and the quantity of residual water in the core; as regards the latter, observations established that the optimum conditions for capillary displacement of petroleum from the cores were realized when the residual water content in the core represented 20–25% of the volume of the pores. Decrease in the velocity of the capillary penetration of the water into the core over a period of time is linked, in the author's view, with the formation of a water-petroleum emulsion in the porous parts of the cores.

M. V. Filinov
Courtesy Referativnyi Zhurnal, USSR

5127. Gimatudinov, Sh. K., and Kusakov, M. M., The influence of the velocity of the displacement of petroleum by water from natural cores on the yield of petroleum (in Russian), *Trudi Mosk. Neft. In-ta* no. 22, 207–216, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12135.

An experimental study was carried out of the influence exercised by the velocity of the displacement of petroleum from natural cores on the final yield of petroleum and on the yield of petroleum in the period when water was absent. The experiments made covered two series: (1) the displacement of petroleum by 0.1% solution of Na_2CO_3 from samples of sand cores, containing 20, 25 and 40% of the volume of pores of residual water at different velocities for the passage of the water; (2) the influence on the petroleum yield of the speed of displacement of the petroleum by natural stratum water from samples containing 20 to 25% of residual water. The results of the experiments are given in the form of graphs showing the relation of the petroleum yield to the velocity of the passage of the water-petroleum mixture. Experiments established that the velocity of displacement of petroleum by water from a porous medium influences in different ways the yield of petroleum, in relation to the quality of the displacing waters, the character of the distribution of the petroleum and water in the porous stratum and the physico-chemical properties of the whole system.

M. V. Filinov
Courtesy Referativnyi Zhurnal, USSR

5128. Arkhangel'skii, V. A., Analysis of axisymmetrical inflow of oil-gas mixture by edge-water drive (in Russian), *Inzhener. Sbornik Akad. Nauk SSSR* **29**, 98–105, 1960.

Paper uses the method of finite differences for determining the rate of oil flow, the rate of gas flow, the pressure and the oil pore saturation. It is shown first that no matter what boundary condition is physically possible on the interface between oil and water, this condition may be always reduced to the knowledge of pressure values on this interface. For calculations pertaining to the flow of the gas-oil mixture, author proceeds from the well-

known Muskat equations, to which he applies finite differences. The method involves however some approximations which concern certain physical parameters entering into formulas, such as for instance the oil saturation which is assumed to vary linearly both in the time interval Δt and in the spatial interval Δr. Since the rate of oil flow and the gas-oil ratio vary slowly in space during the displacement process, a steady flow with given rate of oil flow and oil-gas ratio, corresponding to the moment at which the displacement of the interface begins, can be taken as an initial state. T. D. Oroveanu, Roumania

Geophysics, Hydrology, Oceanography, Meteorology

(See also Revs. 4924, 4927, 5076, 5087, 5092, 5109, 5112)

Book—5129. Applied geophysics [Prikladnaya geofizika], Vol. 24, Leningrad, Gosudarstvennoe Nauchno-Tekhnicheskoe Izdatel'stvo, Neftyanoi i Gorno-Toplivnoi Literatury, Leningradskoe Otdelenie, 1960, 261 pp. 13 r. 25 k.

This book is a series of articles on advances in techniques or in the interpretation of data in the following fields of geophysical prospecting: seismology, electrical resistivity, gravimeter and magnetic surveys and the electrical method known as "well-logging" in U.S.

The contents are indicated by the following brief resume.

Chap. 1. Slutzkowski, A. I. On the efficiency and resolving power of band pass filters used in low frequency seismic amplifiers. The author discusses the advantages of including the filter in the feedback loop and presents graphs which will be of use to the worker in this field.

Chap. 2. Zabyalov, V. D. On the interpretation of seismograms in zones of interference caused by irregular stratification the author establishes the wave fronts by a detailed analysis of seismograms and correlates these with a possible structure.

Chap. 3. Krolenko, N. G. and Tzekov, G. D. On the apparent resistivity of dipping beds. A resistivity technique is developed, the surface of separation being assumed plane. The results have been calculated for angles of dip which are submultiples of 360° and for various resistivity ratios. The results are exhibited as a series of graphs.

Chap. 4. Klushin, I. G. A transformation treatment of gravity anomalies. The author considers a stratum of limited depth and expresses the gravitational anomaly as an integral. By the use of integral transforms he derives a possible source distribution of disturbing matter. Though the results are not unique it is claimed that they may be useful when correlated with other geophysical or geological data.

Chap. 5. Yelanski, L. N. and Pavelyev, S. V. On the second derivative method of analysis of gravity anomaly maps. A finite difference approximation is used and the map is divided into squares. A process of subdivision is used, the appearance of fine detail indicating the approximate depth of the sources of disturbance. An estimate is made of the probable error of the calculation.

Chap. 6. Filinov, E. M. An investigation of the scattering of homogeneous gamma rays in rocks in the density range 1–3 gm/cm. The problem is considered first in an infinite layer of uniform density. Later the scattering problem is considered in the case of a two layer system in which the surface of separation is a cylinder.

Chap. 7. Kulinkovitch, A. E. This chapter reviews the electrical method called "well-logging" (in U.S.). The results are presented in a series of graphs which cover a wide range of resistivity ratios and linear dimensions of the apparatus.

Chap. 8. Sochranov, N. N. The author discusses the problems arising in the method described in Chap. 7 when the bore passes through the transition zone between soil-filled and water-filled strata.

Chap. 9. Borisov, A. A. An account of the geophysical methods used in the preparation of a regional structure map of Turkmenistan. In addition to gravimeter and magnetometer surveys, the methods of seismic reflection and refraction shooting were used over the greater part of the regions. It was possible to correlate the observations with the geology and obtain useful interpretations.

Chap. 10. Lozinskaya, A. M. and Mindlin, L. E. This chapter is mainly concerned with setting up a network for exact aerial navigation when carrying out magnetometer surveys. The position of the plane is determined by the intersections of two families of hyperbolas of constant phase provided from two base stations.

Chap. 11. Plosbkov, M. K. On the design of resistance-coupled low frequency amplifiers and band pass filters for use with seismographs.

Chap. 12. Bibliography of papers published by the State Institute for Geophysical Prospecting between 1956 and March 1959.

J. M. Jackson, Scotland

5130. Stauder, W., S waves and focal mechanisms: The state of the question, *Bull. Seismol. Soc. Amer.* **50,** 3, 333–346, July 1960.

Fracture of rock is generally conceded to be the generating mechanism of earthquake waves. Studies of compressional (P) waves recorded at a distance have been successful in defining either a couple or a pair of couples that could represent the source but cannot resolve the ambiguity.

Author reviews the direction of motion of shear (S) waves observed at a distance from a source as a function of the source mechanism. Mechanisms discussed are: simple force, couple, double couple, dipole without moment and dipole with moment. With proper recordings a study of S waves can resolve the ambiguity above. J. T. Wilson, USA

5131. Droste, Sophia, and Teisseyre, R., The mechanism of earthquakes according to dislocation theory, *Sci. Rep. Tohoku Univ., Japan,* (5) *Geophysics* **11,** 1, 55–71, Sept. 1959.

It is assumed that the origin of an earthquake involves two factors. One of these is the shearing stress and the other is the appearance of discontinuities of displacement in the medium. Dislocations of this type were discussed by Burgers [*Proc. Acad. Amsterdam* **42,** p. 293, 1939]. Such a dislocation experiences a force due to the shearing stress. There is thus a potential energy associated with the dislocations. The mutual interaction of two dislocations has been studied and in certain circumstances the force is attractive. Under these conditions dislocations can coalesce and grow. This model is suggested as a possible mechanism for an earthquake. A formula is given for the release of energy at the surface of the earth and is compared with the estimated energy values associated with several well-known earthquakes (Gutenberg-Richter). The agreement is plausible but not strikingly good. Further study on these lines is required.

J. M. Jackson, Scotland

Book—5132. Kondrat'iev, N. E., et al., Channel development [Ruslovoi protsess], Leningrad, Gidrometeoizdat, 1959, 371 pp. $2.80.

This synthetic study contains articles on hydrological and morphological backgrounds of the theory of channel development; origin and development of river valleys; sources of river sediments; horizontal deformations of channels; formation of river valley; river channel peculiarities on the Russian plane; theory of sediment transportation; current kinematics in bends and meandering; morphometric characteristics of channels of plane rivers. A

valuable contribution to river hydrology and morphology. Bibliography contains 187 Russian and 37 foreign references.

S. Kolupaila, USA

5133. Brisbane, A. P., Hydrodynamic characteristics of a high-speed depth recorder developed by the U.S. Navy Mine Defense Laboratory, David W. Taylor Mod. Basin Rep. 1502, 13 pp., Feb. 1961.

Model tests were conducted to determine the hydrodynamic characteristics of a high-speed depth recorder developed by the U. S. Navy Mine Defense Laboratory. The error in indicated depth due to dynamic pressure at the original orifice location was measured and the true static-pressure point on the housing was located. The drag coefficient was determined and dynamic stability of the body was observed to be satisfactory over the speed range from 0 to 30 knots. From author's summary

5134. Ozmidov, R. V., On the rate of dissipation of turbulent energy in sea-currents and on the dimensionless universal constant in the "4/3-power law," *Bull. Acad. Sci., USSR, Geophys. Ser.* no. 8, 821–823, Feb. 1961. (Translation of *Izv. Akad. Nauk SSSR, Ser. Geofiz.* no. 8, 1234–1237, Aug. 1960 by Amer. Geophys. Union, Washington 5, D. C.)

Author reviews the available data on the rate of dissipation of turbulent energy in the sea. He suggests a method for determining ε from the horizontal turbulent diffusion of particles on the surface of water. The article shows that the universal dimensionless constant in the "4/3-power law" has the order of 0.1 and not 1 as was previously assumed. From author's summary

5135. Abbott, M. R., Coriolis effects on the tidal flow in an exponential estuary (in English), *Houille Blanche* **15,** 5, 616–624, Sept./Oct. 1960.

Paper gives a simple approximate treatment of the tidal flow in an estuary of constant mean depth and exponential width, taking the rotation of the earth into account. The frictional resistance is represented by Lorentz's linearization of Chezy's law. It is shown that the maximum difference of level across the estuary for a progressive wave is given by the same formula as for a uniform frictionless channel but, due to a difference of phase, the difference of amplitude can be considerably less. The theory is applied to a section of the Thames Estuary and it is shown that the amplitude difference is only about a third of the maximum difference of level.

From author's summary

5136. Khodakov, V. F., The interlinking of turbulent flows with quiet ones in river beds which suddenly widen (in Russian), *Naucbn. Zap. Mosk. In-ta Inzb. Vodn. Kb-vo* **20,** 198–214, 1958; *Ref. Zb. Mekb.* no. 10, 1959, Rev. 11855.

The three-dimensional junction of turbulent flows with quiet ones is investigated; the phenomenon occurs when the river-bed widens suddenly. The author determines the free-spreading and the merging flow as two new forms of interlinking. The hydraulic jump in space can be determined as a form of linking of the flows only in the case of river beds widening smoothly. The merging flow is investigated in detail. For this case an equation is derived for lateral inflow, enabling a number of practical problems to be solved. Three types of merging flows are established and recommendations are given regarding the determination of the form of linkage and of the type of regime when the flows are present. The author proposes a method of solution for the discard modulus introduced into hydraulics by N. N. Pavlovskii when the upper and lower waters are linked up in conditions of a 3-dimensional problem. M. F. Skladnev

Courtesy Referativnyi Zhurnal, USSR

5137. Mikhailov, V. N., The dynamics of a river current flowing into a reservoir (in Russian), *Trudi Gos. Okeanogr. In-ta* no. 45, 73–90, 1959; *Ref. Zh. Mekb.* no. 10, 1959, Rev. 11863.

An investigation is made on the form in plane of a river stream flowing into a very wide reservoir; it is assumed that the markings for the bottom may rise or fall. In order to solve the problem the author derives an approximate equation for the dynamic equilibrium of the stream, using a calculation procedure proposed by the abstractor, and introduces an assumption regarding the constancy of the momentum on a portion of the stream which is spreading. This assumption, in view of the conditions governing the problem being investigated, does not evoke criticism as it is assumed that the reservoir has unbounded dimensions. In the particular case, because of the negligible forces of friction on the lateral surfaces, the author is able to adopt the known solution by N. M. Bernadskii which is correct for very wide streams. Comparison of the calculations with the data of laboratory observations proved to be satisfactory; the maximum error did not exceed 15–20%; the picture obtained of the spreading current appeared to be a likely one. The paper concludes with some concepts on the form of river-bed formations at the point where the river enters the reservoir and on the influence of these formations on the character of the flow.

I. I. Levi

Courtesy Referativnyi Zhurnal, USSR

5138. Goody, R. M., The influence of radiative transfer on the propagation of a temperature wave in a stratified diffusing atmosphere, *J. Fluid Mech.* **9,** 3, 445–454, Nov. 1960.

An approximate solution is given to the problem of a temperature-wave propagation through a stratified medium which both diffuses and radiates heat. It is a combination of two wave-components whose amplitudes vary with the height from the lower boundary.

An example, using parameters simulating the atmosphere, suggests that analysis of the propagation of the diurnal wave upward from the earth's surface seems to be inadequate provided radiative effects are not considered. H. Arakawa, Japan

5139. Phillips, N. A., On the problem of initial data for the primitive equations, *Tellus* **12,** 2, 121–126, May 1960.

It is first demonstrated that in a forecast with the primitive equations, pure geostrophic or purely nondivergent initial wind fields will result in noticeable meteorological "noise," especially in a baroclinic atmosphere. It is then shown in a linear example that this noise is greatly suppressed if the divergence of the initial wind field is set equal to the divergence implied by the usual geostrophic theory of numerical weather prediction. The absence of noise in Charney's 1955 test computation with nondivergent "balanced" initial data is explained.

Finally, the quasi-geostrophic expansion is used to define initial data for a nonlinear forecast with the primitive equations. Two cases are considered, that where only the geopotential is known from observations, and that where only the nondivergent part of the wind is known from observation. It is shown that the initial tendencies computed from initial data of this kind preserve the assumed geostrophic character of the flow, and thereby make it possible to get more accurate forecasts with the primitive equations. From author's summary by H. Arakawa, Japan

5140. Dubov, A. S., The influence of mountain ridges on the changes of position of cyclones (in Russian), *Trudi Glav. Geofiz. Observ.* no. 76, 40–49, 1958; *Ref. Zh. Mekb.* no. 10, 1959, Rev. 12060.

The theoretical bases regarding the influence of a mountain ridge on the changes of position and the development of cyclones was investigated by I. A. Kibelli [Tr. NIUGUGMS, 1947, seriya 1, (30)]. The author's paper studies the question set in the frame-

work of another prognostical plan [M. I. Yudin, *Doklady Akad. Nauk SSSR* **3**, no. 33, 1957]. The expression for the nongeostrophic components of the wind u'' and v'' conditioned by the presence of unevenesses of the earth's surface are determined by the system

$$\zeta^2 \frac{\partial^2 \varphi_u''}{\partial \zeta^2} + a^2 \Delta \varphi_u'' = 0 \qquad \left(\zeta = \frac{p_z}{p_0}\right) \qquad [1]$$

$$\zeta^2 \frac{\partial^2 \varphi_v''}{\partial \zeta^2} + a^2 \Delta \varphi_v'' = 0$$

with boundary condition

$$\varphi_u'' = \varphi_v'' = 0 \qquad \text{with } \zeta = 0$$

$$\varphi \Delta_u'' = \frac{g}{p_0} \frac{\partial}{\partial x} q w_\Gamma \quad \text{with } \zeta = 1$$

$$\Delta \varphi_v' = \frac{g}{p_0} \frac{\partial}{\partial y} q w_\Gamma$$

Here φ_u'' and φ_v'' are sought functions by means of which u'' and v'' respectively are determined in a well-defined manner, x and y are horizontal coordinates, p_z is the pressure at the z level, p_0 is the standard pressure at sea level, g is the acceleration of the force of gravity, q is the density, a the constant parameter characterizing the mean stratification of the atmosphere, w_Γ is the vertical velocity of the wind produced by the slipping of the air over the mountain's surface. By assuming that the averaged values of the sought functions φ_u'' and φ_v'' within the periphery with a radius $r \geq 1000$ KM are equal to zero and by recording the given equations for the points in the triangular network with spacings of 400 km, author obtains two monotypic systems of equations in which the derivatives by the horizontal coordinates are replaced by relations for finite differences. A solution for the latter is then looked for in the form of stepped series. By carrying out extensive computations and by performing a number of simplifications and also by using the value for w_Γ, author obtains expressions for u'' and v'' at mountain level and also expressions for the velocity components of the transpositions of the baric centers C_x'' and C_y'' governed by the unevenness of the relief of the earth's surface.

$$u'' = -a \frac{\partial}{\partial x}(\Phi, H_\Gamma), \qquad C_x'' = -a l^2 \frac{\partial H_\Gamma}{\partial y}$$

$$v'' = -a \frac{\partial}{\partial y}(\Phi, H_\Gamma), \qquad C_y'' = -a l^2 \frac{\partial H_\Gamma}{\partial x}$$

Here Φ is the geopotential, while H_Γ is the height of the mountain, l is Coriolis' parameter. The paper concludes with an evaluation of the additional velocity of transposition of baric centers attributable to the presence of mountains possessing some characteristic features.

V. V. Bykov

Courtesy Referativnyi Zhurnal, USSR

5141. Bushuk, V. I., The leading flow and the direction of the change of position of baric formations (in Russian), *Trudi Leningr. Gidrometeorol. In-ta* no. 8, 3–11, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12076.

It is pointed out that accounts of the determination of the leading flow recorded in the existing literature differ to some extent and only reflect in the most general of its features the basic principles of the change of position of baric formations. Author proposes to investigate the leading flow on the basis of the results of his previous work on the subject [*Meteorol. i Gidrologiya* no. 1, 1953], by treating this flow as the predominant direction of flows in a system of vortex circulation, and demonstrates one of the methods for its determination with the aid of the sum total of the vectors of the geostrophic wind on the chart AT-700 and of the thermal wind according to the chart OF-500/1000. Three schematic plans are put forward for the thermobaric fields, with recommendations regarding the periods of time to be adopted for the investigation in the assigned fields of the centers of cyclones with reference to their situation in regard to the earth in order to obtain the optimum convergence of the prognostic and factual trajectories. Reasons are also indicated for possible errors in the prognostic trajectory arising mainly because of inaccurate analysis of the thermobaric charts. The paper concludes with an example of prognosis by the method described of the track of a cyclone.

K. B. Abramovich

Courtesy Referativnyi Zhurnal, USSR

5142. Burman, E. A., The theory of anticyclone foehn winds (in Russian), *Izv. Akad. Nauk SSSR, Ser. Geofiz.* no. 3, 413–418, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12074.

Note is made of the fact that when analyzing anticyclone foehn winds it is essential to investigate not only the changes in the state of individual particles of the air but also the stratification of the whole moving air mass. A formula is derived for the local change of the relative humidity, consisting of a convection component, dependent on the stratification of the humidity and on the temperature, and of a temperature component, which is proportional to the declination of the vertical gradient of the temperature from the adiabatic. Numerical evaluations show that the temperature contribution to the reduction of the relative humidity during a foehn wind is generally not large (of the order of several per cent) while the convection contribution, determined by the inflow of very dry air from the upper strata, is always the dominant one, more especially at high altitudes and at low temperatures. It is only when inversion is present in the free atmosphere that the temperature component effecting a humidity reduction in anticyclone foehn winds can attain significant proportions. Author holds the view that this type of process is more frequently to be met with, for instance in Siberia and the Antartic. He investigated in detail two examples of anticyclone foehn winds, which were observed at the Elbrussk observatory of the Institute of applied geophysics of the Academy of Sciences of the U.S.S.R. in 1954–1955. The changes of relative humidity in these cases were used to estimate the velocity of the descending motions in the anticyclone, which was equal to 2–4 cm/sec in one case and 5–10 cm/sec in the other, showing satisfactory agreement with other estimates of velocities in anticyclones.

A. Kh. Khirgian

Courtesy Referativnyi Zhurnal, USSR

5143. Sadokov, V. P., Questions regarding the direction taken by position-changing cyclones (in Russian), *Meteorol. i Gidrol.* no. 8, 33–36, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12059.

An attempt is made to obtain qualitative results for the change of position of high cyclones based on the analysis of the equations for hydrodyanmics. It is assumed that the change of position of cyclones takes place in the direction of some fictitious flow which in most cases does not coincide with the direction of the structural contour of the isobaric surface 700 mb usually identified with the "leading flow". Here the author uses the ideas contained in the graphical method of prognosis of the field of the geopotential, assuming as established the derivation of the function $B = \bar{\Phi} + f(l)$, where $\bar{\Phi}$ is the mean value of the geopotential Φ along some periphery with a radius R, while $f(l)$ is a function depending on the geographical latitude. The author considers that when investigating the conditions for the change of position of large-scale vortices of great vertical extent it is necessary to introduce into the investigation some mean field of flows (and not a field of flows at one level) which would be characteristic of the general state of the atmosphere. With this object in view a magnitude is

introduced to represent the potential energy of a single column of air Π

$$\Pi(x, y, t) = \frac{1}{g}\int_{p_0}^{0} \Phi\, dp$$

(g is the acceleration of the force of gravity, p the pressure, x and y the horizontal coordinates, t the time) while the equation for the vortex is intergrated by the variable p with the usual boundary conditions. The equation obtained (Poisson's equation) is solved relative to the unknown function $\partial\Pi/\partial t$. Assuming that on the periphery C with radius R

$$\frac{1}{2\pi R}\int_{C_R} \frac{\partial \Pi}{\partial t}\, dC = 0$$

the author introduces a quasiconstant function by time

$$B^\circ = \int_{p_0}^{0} B\, dp$$

on the basis of the last correlation, where B is the function referred to above, introduced originally by N. I. Buleev. The field B° corresponds to the mean field of flows and, as pointed out by the author, high baric systems should change position in the direction of the lines B° = const. The author plotted 30 charts of the field B°. It is demonstrated that comparison of the tracks of cyclones having directions of the isolines B° = const gave satisfactory results. The average divergence of the trajectories of cyclones from the line B° = const was 200 km for a 1000 km long track. Such divergencies can be explained by the fact that during the investigation of the change of position of cyclones the advection inflow of warmth is not taken into account. Some examples are given.

V. V. Bykov

Courtesy Referativnyi Zhurnal, USSR

5144. Uspenskii, B. D., The congruence between the vertical velocities, the isallobars and the isallogypses of an absolute topography (in Russian), *Trudi Tsentr. In-ta Prognozov* no. 70, 71–73, 1958; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 12063.

Simplified formulas are derived for the calculations of the vertical velocities (or more accurately—the individual changes of pressure representing in themselves the analog of velocities in the so-called isobaric system of coordinates) on the principal isobaric surfaces. These formulas were obtained by combining the equations for continuity with the equations for vortexes. The latter are utilized in a simplified form, which takes into account the correlational linkage between the local variations in the geostrophic vortex, on the one side and the geopotential on the other. This is the form for the equations of vortexes adopted by the author in previous works (for instance, *Meteorol. and hydrology* no. 5, 3–8, 1954. Formulas are eventually obtained, after making some additional simplifications, which express the vertical velocities through the local changes of pressure p_0 at sea-level and at an altitude of H_γ of the isobaric surface 700 mb, through the geostrophical advection of the vortex at these levels and the advection p_0. The formulas have the form

$$\begin{aligned}
w_9 &= 4.0\,\partial p_0/t - 0.06\,(p_0, \Delta p_0) + w_{p_0}{}' \\
w_8 &= 5.9\,\partial p_0/\partial t - 0.09\,(p_0, \Delta p_0) + w_{p_0}{}' \qquad (6) \\
w_7 &= 8.6\,\partial p_0/\partial t + 3.5\,\partial H_\gamma/\partial t - 0.13(p_0, \Delta p_0) - 0.07\,(H, \Delta H)_7 + w_{p_0}{}' \\
w_5 &= 8.0\,\partial p_0/\partial t + 12.5\,\partial H_\gamma/\partial t - 0.13\,(p_0, \Delta p_0) - 0.28(H, \Delta H)_\gamma + w_{p_0}{}'
\end{aligned}$$

where w_9, w_8, w_7 and w_5 are the analogs of the vertical velocities in mb for 12 hours on surfaces of 900, 850, 700 and 500 mb respectively, t is the time; by Δ is indicated a horizontal Laplacian value and by round brackets, a Jacobian value.

$$w_{p_0}{}' = 0.012\,(u\,\partial p_0/\partial x + v\,\partial p_0/\partial y)$$

where, in their turn, u and v are components of the wind's velocity in km/hr along the axes x and y.

L. S. Gandin

Courtesy Referativnyi Zhurnal, USSR

5145. Fleagle, R. G., Some estimates of the effects of heating on large scale disturbances, *Tellus* **12,** 2, 127–133, May 1960.

Effects of heating on large-scale disturbances are studied by assuming that heating is tuned to the disturbances, with rate of heating proportional to individual pressure change but displaced by an arbitrary phase angle. The linearized two-layer baroclinic model reveals that the heating which accompanies condensation in the middle troposphere may lead to marked increase in instability and to shift of maximum instability from five to three thousand kilometer wavelengths, and this calls attention to a serious defect in the two-layer model which must influence the accuracy of numerical calculations using the nonlinear equations.

From author's summary

Naval Architecture and Marine Engineering

(See also Rev. 4939)

5146. Mandel, P., Subcritical and supercritical operation of ships in waves and the coincidence of maximum damping, *J. Ship Res.* **4,** 1, 22–34, June 1960.

Several papers by Lewis have discussed the relationship between hull form and critical and supercritical operation in ahead seas. The present paper extends consideration to overtaking and following seas and defines subcritical, critical, and supercritical zones of operations in ahead, overtaking, and following seas. A region is pointed out where unorthodox ships may be designed to operate where resonant and near-resonant conditions with all important wave components may be almost entirely avoided throughout a substantial range of speeds in ahead and overtaking seas. In the limited case where resonance is encountered with important wave components in this region, it is shown that maximum wave damping might be simultaneously experienced. In the course of developing this subject, it was found that the theoretical condition for maximum wave damping is identical in overtaking seas with a particularly significant case of resonance.

From author's summary by W. J. Carter, USA

5147. Cumming, R. A., Resistance and EHP of a planing catamaran as predicted from tests of model 4836, David W. Taylor Mod. Basin Rep. 1478, 7 pp. + tables and charts, Nov. 1960.

Model tests were made to determine the powering characteristics for a catamaran-type hull. Wetted lengths, running trims, and resistances were measured on the model for a number of speeds, displacements, and initial trims. In addition, the effect on performance of changes in hull spacing, planing area, and spray rails was determined. A test was also made with a step on the roof of the tunnel. The results are presented in dimensionless form and also in the form of EHP curves for 70-ft, 105,400-lb boat.

The data obtained from the tests indicate that for this hull a wide spacing is of no advantage from a resistance point of view. The spray rails on the final configuration increased the drag slightly on the full-sized boat. The best configuration was not as good as a good conventional planing hull, and this is thought to be due primarily to a large amount of air drag.

From author's summary

5148. Sasajima, H., Yoshida, E., Tanaka, I., and Nakato, M., Form effects on frictional resistance and power prediction of large tankers, *Technol. Rep. Osaka Univ.* **9,** 79–89, Mar. 1959.

5149. Clement, E. P., and Pope, J. D., Stepless and stepped planing hulls—Graphs for performance prediction and design, David W. Taylor Mod. Basin Rep. 1490, 34 pp., Jan. 1961.

This report presents graphs by means of which the high-speed resistance and trim of conventional and stepped planing boats of a wide range of sizes and proportions can be determined. Graphs which give guidance in selecting parameters which will result in optimum planing performance are also presented. Values for the graphs were obtained from equations for the lift, center of pressure, and resistance of prismatic planing bottoms which were previously developed by the National Aeronautics and Space Administration and the David Taylor Model Basin.

From authors' summary

5150. Markiewicz, H., and Wysoczynski, J., A new method for objective measurement of ship velocity (in German), *Jahrbuch Schiffbautech. Gesellsch.* **53,** 310–321, 1959.

A method of measuring ship speed by determining the time for the ship to travel its own length is described. The equipment for recording the time and distance data consists of a reflex camera equipped with a telephoto lens on a tripod and a clock. The image of the ship is recorded through a slit on a continuously moving film while a time scale is recorded through another slit by a light flashing every second, the resulting photograph showing the ship against a time scale. The method is claimed to be accurate to 0.2 per cent.

L. Landweber, USA

5151. Lambert, J. D., The effect of changes in the stability derivatives on the dynamic behaviour of a torpedo, Aero. Res. Counc. Lond. Rep. Mem. 3143, 24 pp., 1960.

Author calculates the sensitivity of a torpedo radius of turn, stability margin, and time constants to changes in the various hydrodynamic derivatives. The linearized expressions for sensitivity are derived and numerical results presented for two representative torpedoes. The analysis considers only uncontrolled motions. Since the sensitivities are quite different in the two illustrative cases, the implications of the results would depend on the hydrodynamic characteristics of the particular vehicle under investigation. As the author points out, this type of analysis may aid in indicating which characteristics should be adjusted to achieve desired changes in dynamic stability. Although not stated in the paper, if the stability is sensitive to the values of some coefficient, it may also be sensitive to errors in the basic equations of motion which are only approximate.

L. H. Schindel, USA

5152. Ochi, K., Model experiments on the effect of a bulbous bow on ship slamming, David W. Taylor Mod. Basin Rep. 1360, 29 pp., Oct. 1960.

This report presents the results of model experiments to determine the effect of a bulbous bow on ship slamming. The experiments were conducted on two 5.5-ft models; one, the MARINER, had a bulb whose area was 4 per cent of the midship area; the other, the modified MARINER, had the same principal dimensions as the MARINER but had no bulb.

Based on the experimental results, a comparative evaluation is made of the effect of the bulb on slamming. It is found that the differences between the MARINER and the modified MARINER are not as large as had been expected.

The MARINER appears to be superior to the modified MARINER at speeds less than 1.5 knots (14.7 knots full scale) and inferior to the modified MARINER at speeds above 2.0 knots (19.6 knots full scale).

From author's summary

5153. Strom-Tejsen, J., Damage stability for ships, *Ingenioren, Copenhagen* **5,** 1, 16–24, Mar. 1961.

A systematic damage stability calculation has been carried out by means of the digital computer DASK. The results of the calculation are given in diagrams, which make it possible to read the required stability which will enable the ship to withstand the final stage of flooding of a damaged compartment. In the case of symmetrical flooding the minimum metacentric height required for the initial condition is given by the equation

$$GM_{min} = d \cdot (f_M + f_I) + 0.05 \text{ meters}$$

where d is the initial draught, and f_M and f_I are read from the diagrams dependent on the position of the compartment, the height of double bottom, etc.

In the case of unsymmetrical flooding a further correction is necessary.

The diagrams are applicable to the preliminary design stages for a passenger ship, and make it possible to estimate the effect of modifications to the damaged compartment. Because of the uncertainty of the diagrams, they should not be used in place of an exact damage stability calculation.

From author's summary

5154. Lum, S. M. Y., and Folb, R., Evaluation of the static yawing stability of the AN/SQS-22 sonar vehicle with three different tail configurations, David W. Taylor Mod. Basin Rep. 1488, 12 pp., Jan. 1961.

The static yawing stability of the AN/SQS-22 sonar body with three different tail configurations, designed by the U. S. Navy Underwater Sound Laboratory, was determined at the David Taylor Model Basin. Basin tests indicate that one of the three proposed configurations is statically stable, one marginally stable, and one unstable. Theoretical approximations of the yawing stability tend to confirm the experimental data.

From authors' summary

5155. Beveridge, J. L., Propulsion characteristics of a submerged model as affected by Reynolds number, David W. Taylor Mod. Basin Rep. 1454, 13 pp., Dec. 1960.

The results of an experimental investigation on the effect of Reynolds number on model propulsion tests of a submerged body are presented. A propeller was characterized in open-water and behind a 15-foot submerged body of revolution. The influence of test Reynolds number on propeller thrust and torque is determined and the results obtained are discussed.

From author's summary

5156. Arshava, A. M., Flexible shells—a new type of cargo ships (in Azerb.), *Izv. Akad. Nauk AzerbSSR* no. 12, 39–51, 1957; *Ref. Zh. Mekh.* no. 10, 1959, Rev. 11793.

The possibilities are discussed of transporting liquid products by sea in specially built flexible shells, and examples are given for such use of flexible shells. Experiments are described, carried out by the author with models to find out the sea-going qualities of flexible shells filled with liquid loads. One such model in the form of a bag made from cotton fabric impregnated with a drying oil had a length of 1580 mm. In transverse section the model was oval with the large diameter measuring 460 mm and the small ~200 mm. Inside the bag were three partition walls with valves which could be closed. The shell had a capacity of 60 liters of liquid with a specific gravity of 0.9. The specific weight of the model was 2.33 kg. Observations on the model were made with a wind of 5–6 m/sec, a height of the waves of 0.8 m and a towing rate of 5 m/sec. Another series of tests was made with a shell manufactured from polychlorvinyl with a length of 2500 mm and a width of 500 mm. Fuller details of this type of shell are not given

in the paper. The third series of tests was carried out with a shell of a two-layer diagonally folded rubberized fabric with a length of 5000 mm and a width of 1000 mm. The shell could take 500 liters of liquid with a specific gravity of 0.73. The tests were made with a towing speed of 1 m/sec, the height of the waves was 1–2 m, the steepness of the wave 1/20, the course angle 0°–180°. The tests disclosed good sea-going data for all the three models, on the strength of which the author concludes that the problem of perfecting transport ships in the form of flexible shells "was not as lacking in promise as might have been thought in the beginning." An attempt is also made in the study to investigate theoretically the behavior of the shell and its content of a liquid mass when subjected to wave action.

I. I. Vorovich
Courtesy Referativnyi Zhurnal, USSR

5157. Anderson, B. J., On planing boats, Trans. Roy. Inst. Technol., Stockholm 141, 29 pp., 1959.

Friction, Lubrication and Wear

(See also Revs. 4635, 4789, 4831, 4834, 4835, 4852, 4943)

5158. Spurr, R. T., Some measurements of rail-type adhesion, *Wear* **3,** 6, 463–472, Nov./Dec. 1960.

The adhesion between the wheels of a motorized bogie and the rails on which it runs has been measured under a variety of conditions. The effect on adhesion of oil, water and other contaminants has been examined and a number of materials including abrasives, detergents, and silicones, have been tested in attempts to increase the adhesion. From author's summary

5159. Peleg, M., Road friction tests in Israel, *Bull. Res. Council Israel* **8** C, 3, 81–88, Oct. 1960.

Twenty-six points on the main roads of Israel were examined with regard to their friction properties. Using the Leroux "rugosimeter", the percentage was determined of kinetic energy of a pendulum absorbed by friction between its rubber lining and the road surface under dry and wet conditions. The safety coefficient of the road was calculated and the category of allowable speed determined.

Observations have shown that for most of the roads tested only low and moderate speed driving is safe. It was found that the maximum aggregate size to be used for asphalt wearing courses, in order to obtain the highest safety coefficient, should not exceed ¾ in. It was also found that, contrary to the common view, cement concrete roads become slippery when wetted and their safety coefficient is not high. From author's summary

5160. Buckley, D. H., and Johnson, R. L., The influence of silicon additions on friction and wear of nickel alloys at temperatures to 1000 F, *ASLE Trans.* **3,** 1, 93–100, Apr. 1960.

Small additions of silicon have considerable beneficial effect on performance of slider alloys. This effect has most frequently been attributed to increased hardness. The research reported was conducted to consider a hypothesis that the primary role of silicon in slider alloys is one of supporting the formation of protective surface films.

Friction and wear data were obtained at temperature from 75 to 1000 F with a series of binary silicon-nickel alloys containing up to 10% silicon. Pertinent hot hardness, metallurgical and surface-film analysis data are included. Atmospheres used were air, mixtures of oxygen and argon, a mixture of hydrogen and nitrogen, and a halogenated methane gas lubricant.

The results show the role of silicon as a film former to be of great importance in success of silicon-containing alloys as slider materials for extreme temperatures. The range of variables studied gave friction coefficients from 0.05 to > 10.00 depending on film formation tendencies. Alloys with 5% or more silicon having a duplex structure showed the best results. Film formation resulted from surface reactions or the smearing of the softer phase from the alloys having duplex structure.

From authors' summary

5161. Selby, T. W., A low-temperature study of the relationship between engine cranking speed and lubricant viscosity, *ASLE Trans.* **2,** 2, 208–216, Oct. 1959.

The viscosity of motor oils at low temperature strongly affects the cranking speed of the motor. In a previous paper the author has shown that the viscosity characteristics of motor oils at low temperatures may be unusual, especially when they contain Viscosity Index Improvers.

In this paper the author has analyzed the results of 250 cranking tests on two different V-8 engines conducted at temperatures from +3 to – 35 F, using three fluids, two of which contained Viscosity Index Improvers. The data seem to indicate an exponential relationship between the cranking torque M, and the cranking speed, N, and viscosity, μ, such that $M = c(\mu N^b)$ in which b and c are constants and $b \simeq 0.5$. Further results of this study strongly indicate that the correlation is significantly better between cranking speed and viscosities measured at shear rates of approximately 2000 sec^{-1} than the correlation between these cranking speeds and viscosities determined by the ASTM extrapolation technique (Walther equation), or by a recent empirical modification of this technique. From author's summary

5162. Boes, D. J., The solubility of helium in various lubricating oils at high temperatures and pressures, *ASLE Trans.* **3,** 2, 232–236, Oct. 1960.

The helium solubility characteristics of six low-viscosity lubricating oils have been investigated; namely, diester, aryl phosphate ester, dimethyl silicone, methyl phenyl silicone, paraffin-base oil, and aromatic-base oil. Solubility measurements were made over a 1000 psi pressure range and a 350 F temperature range. Results show that the solubility of helium in all fluids studied increases linearly not only with pressure but also with temperature. All fluids except the silicones exhibited heavy foaming tendencies upon decompression.

From author's summary

5163. Nelson, R. K., and Valentine, L. J., High horsepower and high torque performance of EP gear lubricants, *ASLE Trans.* **3,** 2, 237–243, Oct. 1960.

In recent months a series of premature gear failures in field-operated vehicles produced indications of insufficient load-carrying qualities in extreme pressure lubricants of the GL-4 specification conforming to the new Military L-2105A. This condition prompted dynamometer testing in an investigation of horsepower absorption and high torque absorption of these lubricants as compared with extreme pressure lubricants conforming to Military L-2105 specifications.

The results of the dynamometer testing showed that lubricants conforming to the new Military L-2105A were capable of withstanding continuous high horsepower applications where relatively low torque was required, but were not able to compare favorably with the extreme pressure lubricants of the Military L-2105 specification in regard to continuous high torque applications. The gear-tooth surfaces on such tests with lubricants conforming to the new Military L-2105A were severely scored, whereas the Military

L-2105 lubricants produced a satisfactory tooth condition under identical testing conditions. From authors' summary

5164. Barcroft, F. T., A technique for investigating reactions between E.P. additives and metal surfaces at high temperatures, *Wear* **3,** 6, 440-453, Nov./Dec. 1960.

A method is described for studying the reactions between Extreme Pressure (E.P.) additives and metal surfaces at temperatures up to 600 °C. A thin metal wire is heated electrically while immersed in an E.P. oil. Wire temperatures are obtained from a knowledge of the resistance/temperature characteristics of the metal used. The rate and extent of reaction between metal and E. P. additive can be deduced from the change in resistance of the wire with time. Typical results illustrate the applicability of the method to the study of the mechanism of action of E.P. additives. From author's summary

5165. Kestens, J., Stability of shaft position in a hydrodynamically lubricated bearing (in French), *Wear* **3,** 5, 329-357, Sept./Oct. 1960.

Careful mathematical analysis is made of stability criteria for 360° journal bearings of infinite length, based largely on work of N. Tipei "Hidro-Aerodinamica Lubrificatiei," Ed. Acad. Rep. Popul. Romine, 1957]. For the Sommerfeld solution, based on constant viscosity and unbroken oilfilm (negative pressures in divergent sector of clearance space), author finds: (a) under constant load, shaft can be in equilibrium if there is no velocity or acceleration of center; (b) under constant load, small displacements from equilibrium result in instability; (c) for mass-less rotor, stable elliptical motion occurs at approximately half shaft speed. For the Tipei solution, based on zero pressure in divergent region and on viscosity function of temperature and pressure, author finds: (a) stability under contsant load; (b) stability when eccentricity ratio is greater than 0.4 or less than 0.175 for an infinite bearing. Tipei results are found to be close to experimental experience. D. F. Wilcock, USA

5166. Licht, L., and Elrod, H., A study of the stability of externally pressurized gas bearings, *ASME Trans.* **82** E (*J. Appl. Mech.*), 2, 250-258, June 1960.

Paper deals with an important question in modern bearing theory, i.e. with the stability of pocketed thrust bearings. While the first model consists of an infinitely long rectangular strip having a recess area along the line of symmetry, the second bearing model has a circular recess-pocket centered about the supply nozzle.

Assuming laminar flow and isothermal change of state of the lubricating gas within the annulus region, the simplified Navier-Stokes equations are treated by use of a perturbation procedure. After transforming the basic relations by use of the Laplacian—due to the complexity of the problem—the authors focus attention upon the limiting values of certain bearing parameters rather than upon closed-form solutions for the bearing behavior.

Authors give recommendations to insure stability of a pocketed thrust bearing. Reviewer believes that a number of the recommended extremum requirements—e.g. the depth of pockets for stability to be minimized—are boundary extrema rather than extrema located within the range of practical design.

A. Slibar, Germany

5167. Raynor, S., and Charnes, A., Flow parameters in hydrostatic lubrication for several bearing shapes, *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 257-264, June 1960.

Paper analyzes the motion of a liquid between two parallel surfaces and supplied through a hole, with applications to hydrostatic bearings. The basic idea consists in the fact that since the pressure is in such a case a harmonic function, an analogy may be established with the plane potential motion of an incompressible fluid. In this manner the authors refind the results known for the case of two circular disks centrally supplied (similar to the motion due to a source) and by introducing certain correction functions and conformal mapping, the motion is determined for the case of rectangular and elliptical surfaces, or for narrow infinite strips supplied through a central hole.

It is worth noting that the results obtained may be extended to gas-lubricated hydrostatic bearings, a fact which is not mentioned by the author. Moreover, other cases corresponding to known problems in fluid mechanics may be studied in this manner, e.g., annular surfaces or infinite strips supplied through row of holes, rigorous solutions for rectangular plates with the aid of elliptical functions, etc. As far as the reviewer knows, some of these problems have already been examined, e.g., in Kotliar's papers (see Ia. M. Kotlian, "Tecenia viaskogo gaza v zazore mejdu dvumia koaxialnimi tzilindriami," *Izv. Akad. Nauk SSSR, Otd. Tekn. Nauk* no. 1, 12-18, 1957; "Nekatorie primeri dvijeniz viaskogo gaza v uzkom zazore peremennoi tolscini," *Izv. Akad. Nauk SSSR, Otd. Tekn. Nauk* no. 5, 34-39, 1958; "K teorii vozdusnih podvesov sfericeskogo tipa," *Izv. Akad. Nauk SSSR, Otd. Tekn. Nauk* no. 6, 21-26, 1959).

V. N. Constantinescu, Roumania

5168. Jones, A. B., A general theory for elastically constrained ball and radial roller bearings under arbitrary load and speed conditions, *ASME Trans.* **82** D (*J. Basic Engng.*), 2, 309-320, June 1960.

This paper is a bold endeavor to solve by numerical methods the very complex problem of the internal motion in a ball or roller bearing through a minimum number of simplifying assumptions. Proceeding from certain previous results, author writes the motion equations assuming the existence of five degrees of freedom. Both the outer and the inner rings of the ball bearing and its support are considered as being elastic. Besides the external loads, the centrifugal forces and the gyroscopic moments acting on the moving elements of the ball bearing are considered. The five equilibrium equations are linearized and iteratively solved by the Newton-Raplason method, the calculations being programmed for an IBM-704 digital computer. Particularizations are made both for static loads and high velocity conditions and for various conditions of clamping the outer ring. Finally the problem of estimating the ball-bearing life by using Lundberg-Palmgren theory is examined. No numerical example is included, although this would have helped the reader.

Author is to be congratulated for approaching such a complex subject which is of particular practical interest. It is desirable however that the investigations be continued in view of obtaining final results plotted in diagrams which might be used by designers. Moreover, although many approximations currently used are eliminated, reviewer believes that, in order to obtain results in close agreement with experimental data, in addition to the rolling motion of balls and rollers, the sliding motion due to the existence of a lubricant film must be also considered. Indeed this latter motion modifies not only the ball-bearing life but also the distribution of forces and deformations at various points. (A separate study of the hydrodynamic lubrication of a ball or roller has been already made; see, for instance, N. Tipei, "Hidro aerodinamica lubrificatiei (Hyrdo-Aerodynamics of Lubrication) Editura Acad. R.P.R. 1957.) V. N. Constantinescu, Roumania

5169. Muenger, J. R., and Derby, N. C., Two measurement techniques used in the evaluation of cutting fluids, *ASLE Trans.* **3,** 1, 55-60, Apr. 1960.

Various criteria have been used in metal cutting operations to study the effectiveness of cutting fluids; for example, tool-life, wear, temperature and forces; work surface finish; and chip deformation. In each case a number of measurement methods may be employed. This paper describes two very simple means of measurement which have been found useful in cutting fluid research. The first is based on measurement of work surface finish in the direction of cut by determing the amount of pencil lead abraded by the surface under controlled conditions. The second is based on measurement of chip deformation by electrical resistance determination of chips from a known depth of cut.

From authors' summary

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